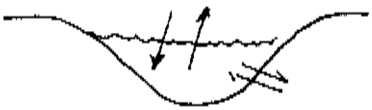
Heat Budget for a Water Body

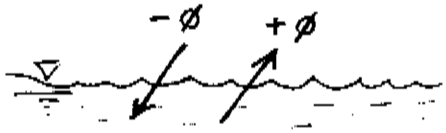
Think of a water body as a control volume with fluxes of energy in and out.



Define thermal energy fluxes by the symbol ϕ (phi) with units of J m^{-2} s^{-1}. A heat balance equation for the water body can be defined by:

$$\phi_{net} = \Sigma[\phi_1 + \phi_2 + \phi_3 + \cdots \phi_n]$$

where the component fluxes may have either + or − signs corresponding to outputs and inputs. A common convention is POSITIVE = OUT, NEGATIVE = IN. (This is just a sign convention, there is no particular reason why).



Using this convention, a value of $\phi_{net} > 0$ corresponds to a water body that is *cooling*, whereas $\phi_{net} < 0$ means the water body is *warming*. So, what are the major component fluxes (ϕ_i)? First, consider radiative fluxes.

RADIATIVE HEAT FLUXES

There are three key radiative flux components, corresponding to three types of radiation:

ϕ_s = shortwave in = I_s^*
ϕ_l^* = longwave in = I_l^*
ϕ_l = longwave out = I_l

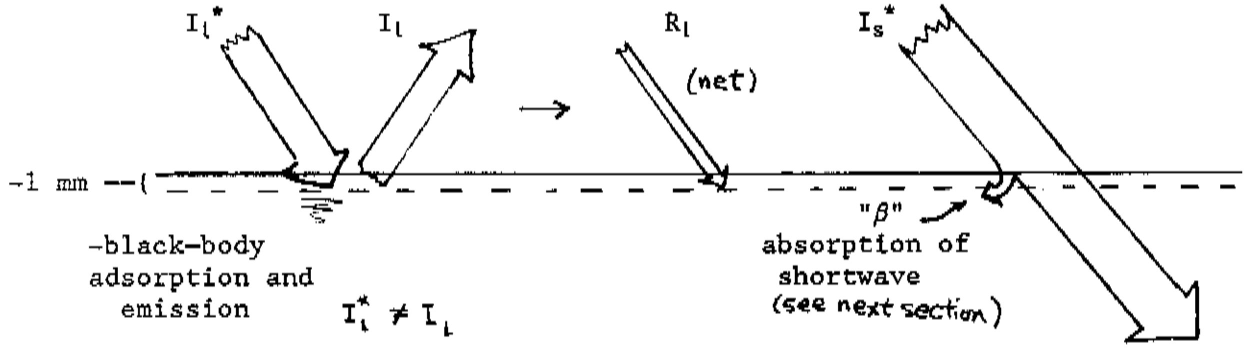
We switch from "I" to "ϕ" in order to emphasize that we are concerned now with the thermal energy fluxes that result from radiation, not with the radiation itself. (ϕ stands for any heat flux, not just radiative). In a problem set exercise we will see how these three fluxes can be estimated for a particular environmental condition.

1. Longwave Radiation

Water is such an efficient absorber of infrared light that longwave radiation cannot travel a significant distance through the water column. A thin film of water at the interface absorbs nearly 100% of all incoming longwave radiation (I_l^*). For the same reason (remember Kirchoff's Law), the water surface is a very efficient grey-body radiator with emissivity $\varepsilon_w = 0.97$ (i.e., very near a perfect black body). The longwave backradiation from the surface is thus $I_l = 0.97\sigma T_s^4$. If longwave radiation is entering and emanating from the same, very thin plane of water, it may be convenient to describe the net (or "effective" longwave radiation (R_l) at the surface. Eagleson's chapter⊛ mentions this and contains the equation:

$$R_l = I_l^* - I_l \quad \text{(net longwave = atm.lw. – water lw.)}$$

I am not sure I see why there is a particular advantage to this, other than conceptually, because you are going to be measuring or calculating the component fluxes anyway. Also, the equation as it appears in Eagleson is confusing as to the signs. If we use a consistent sign convention for fluxes, then I_l^* will always be negative (an input) and I_l will always be positive (an output), so this equation is better written as $R_l = I_l^* + I_l$.



2. Shortwave Radiation

The shortwave flux ϕ_s is a bit more complex because the incoming light is absorbed throughout the upper reaches of the water column. As discussed in greater detail in the Eagleson chapter⊛, light penetration into the water column is gradually attenuated by absorption. The attenuation is quantified by Beer's Law (now generally known as the Lambert-Beer Law in honor of the major contribution of Lambert) which states that

$$dI_\lambda = -a_\lambda I_\lambda$$

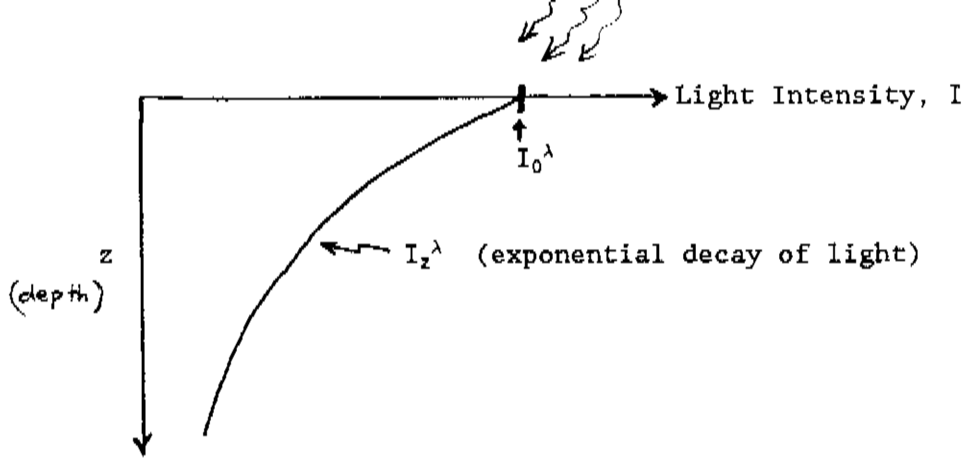
where a_λ is the extinction coefficient for light of wavelength λ. (The symbol η is also commonly used instead of a). This differential simply expresses the idea that each infinitesimal layer of water (or any other medium) absorbs a constant proportion of light. E.g., if $a_\lambda = 0.20$, then each infinitesimal layer absorbs 20% of the light it receives; the change in intensity is a constant proportion of the intensity. (This is just like compound interest on a bank account, only it is a loss rather than a gain). When you integrate across all the differential elements, you obtain an exponential decay law (just like the exponential growth law you obtain for compound interest).

$$I_z^\lambda = I_0^\lambda e^{-a_\lambda z}$$

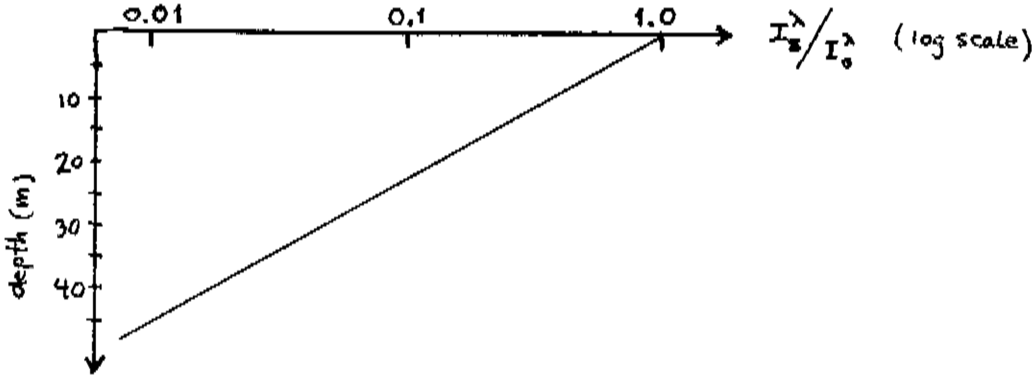
or

$$\frac{I_z^\lambda}{I_0^\lambda} = e^{-a_\lambda z}$$

"EAGLESON CHAPTER"
⊛ See Appendix
at end of this file
for a summary of
this information



The exponential form of the Lambert-Beer law can be linearized by taking the logarithm of both sides, or by plotting the light intensity on semi-log scaling.



However, sunlight is not monochromatic and each wavelength has its own extinction coefficient. Red and orange are absorbed very quickly, whereas shorter wavelengths are less absorbed. The figure below shows this wavelength-dependent attenuation. This phenomenon explains why pure water looks blue, and why all colors except blue disappear below a few meters depth (most underwater color photography requires a local light source like a flash).

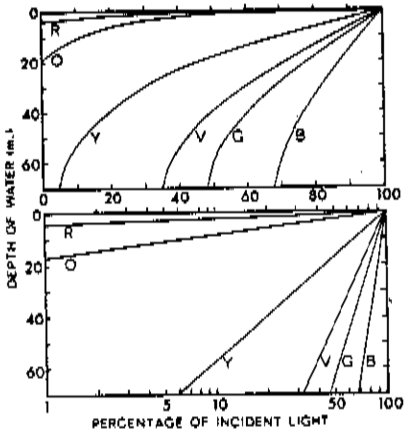


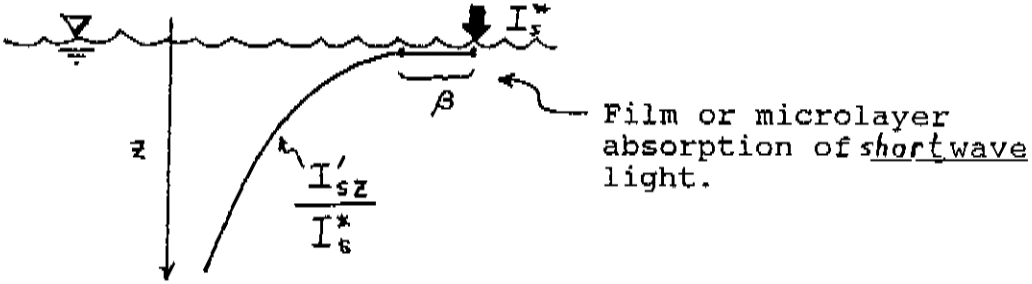
Figure 5–7 Transmission of light by distilled water at six wavelengths (*R*—720, *O*—620, *Y*—560, *G*—510, *B*—460, *V*—390 nm). Percentage of incident light that would remain after passing through the indicated depths of water expressed on a linear (*upper*) and a logarithmic (*lower*) scale. (After Clarke, 1939.)

Although this differentiation of wavelength with depth is important when considering photosynthesis or other photochemical processes, it is unimportant if all we are interested in is heat flux. A single, <u>average</u> extinction coefficient should be adequate.

Dake and Harleman (among others), as cited in Eagleson, calculated average absorption (extinction) coefficients by integrating across all wavelengths of sunlight. They obtain an equation of the form:

$$\frac{I'_{sz}}{I^*_s} = (1-\beta)\,e^{-K|z|}$$

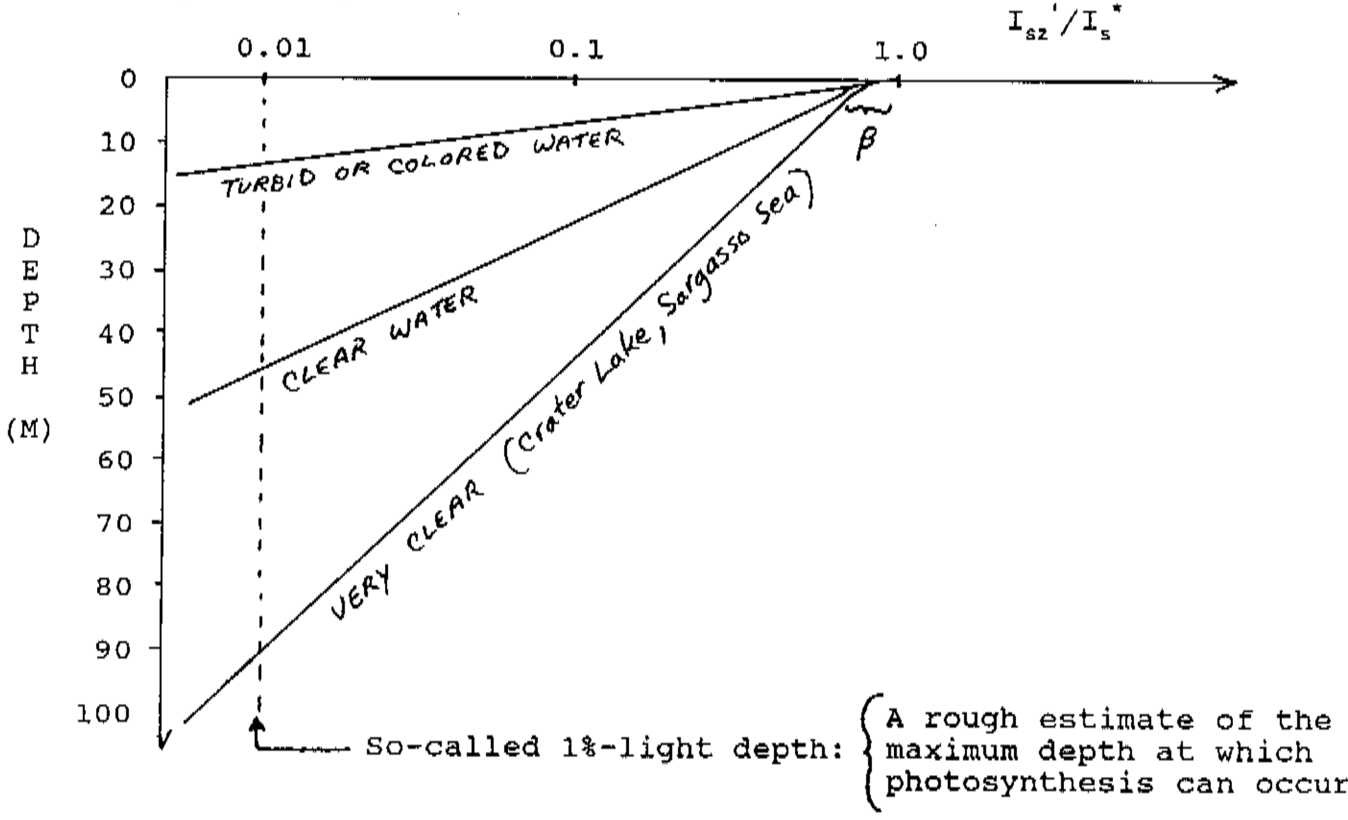
where:
I_s^* - net shortwave light incident upon horizontal water surface
I_{sz}' - same propery at depth z in water column
K - empirical, weighted average extinction coefficient
β - shortwave light absorbed surface film or surface microlayer



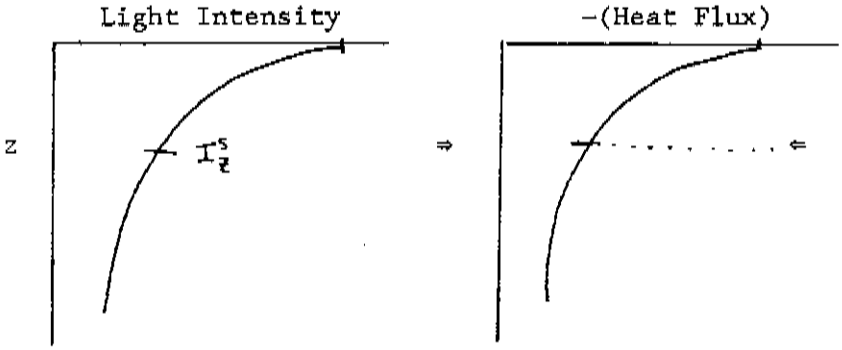
(I suspect that β may "fudge" some of the strong absorbance of red light in the first few centimeters.)

Absorption of Light by Water: Logarithmic Presentation:

The exponential equation given by Beer's Law can be linearized by taking the natural log of both sides, or, by plotting light intensity on semi-log scaling.



Because the shortwave light is absorbed throughout the upper water column, the associated heat flux is distributed across this region, in proportion to the Beer–Lambert Law.



E.g., this layer of water receives ϕ_z^s where ϕ_z^s is proportional to I_z^s

This vertical distribution of heat flux will become critically important later in this course when we study the generation of buoyant water by penetrating radiation. This turn out to be a somewhat tricky problem. For our current purpose of an overall heat budget, we can assume that <u>all</u> of the shortwave solar input is absorbed somewhere in the water column, and therefore all we need to worry about is $\phi_s = I_s^*$. Hence the net radiation flux is given by:

$$\phi_r = \phi_s + \phi_l + \phi_l^*$$

That is, ϕ_r is the radiation component in our computation of the overall net heat flux. A typical range of values for ϕ_r is -1000 J $m^{-2}s^{-1}$ (for a water surface receiving intense solar radiation) to $+400$ J $m^{-2}s^{-1}$ (for a warm water surface on a clear night).

Be cautious about the numerous uses of the word "net" in this discussion (and in other notes, articles, or textbooks). The "net radiation" flux is the net of the inputs and outputs of radiation only; it is not the final "net" heat flux. You will see similar uses of "net" in discussions of atmospheric longwave radiation. The use of "net" is helpful, but it can also be very confusing. Did the author mean the "net flux" or the "net net flux"? If you are confused about any such usages in class, be sure to ask me to clarify.

HEAT FLUXES ASSOCIATED WITH MASS FLUXES

1. **Bulk Water Fluxes:**

The thermal energy borne in and out by river flows or groundwater seepage is important in small basins with short residence times. A good example would be a small lake directly downstream from a deep, dammed reservoir. The reservoir probably discharges cold bottom waters which could dominate the thermal behavior of the small downstream lake. Usually, however, water inputs and outflows are negligible heat fluxes for natural lakes.

Sometimes rain has a small localized effect of the temperature of surface water. Interestingly, rain may be more important than river flows because the effect of rain is concentrated right at that critical air–water interface, and can affect the whole surface of the interface simultaneously. If rain lowers the temperature of that interface by 1–2°, there may be a distinct (albeit temporary) perturbation of the net heat flux across the interface.

2. Evaporation or Condensation:

This is the big enchilada. Evaporation can carry a huge amount of heat away from a water body. Consider that each parcel of mass leaving or entering the water by evaporation or condensation carries heat with it in two forms: sensible heat (as did the river flows discussed above) and latent heat. We assume at this point that you have read and understood the introductory notes on the basic thermodynamics of these properties.

$$H_T = H_L + C_V T_s$$

H_T = Total Specific Heat = heat carried per gram of water transferred

H_L = Latent heat per gram ≈ 2,400 J/g

$C_V T_s$ = Sensible heat per gram ≈ [4 J/g/°C][Surface Temp.]
↑
0° – 35°C

Hence $H_L \gg C_V T_s$, and $H_T \approx H_L$. So for a mass flux ϕ_m due to evaporation or condensation, we can approximate the associated heat flux by:

$$\phi_e = \phi_m H_L$$

Typical values of ϕ_e range from -100 J $m^{-2}s^{-1}$ for strong condensation at the surface up to $+1000$ J $m^{-2}s^{-1}$ for strong evaporation. These numbers reflect the fact that condensation can be significant heat input to a lake, but it never attains the magnitude of the heat loss that is possible via evaporation. Also note that heat flux due to evaporation is often of the same magnitude as the net radiative input.

It would appear that evaporative heat flux is easy to compute once you know the evaporation rate for the lake. The evaporation rate, unfortunately, is very tricky to predict as a function of meteorological conditions. And, it is just about as difficult to measure it directly as it is to predict it. Direct measurement can be made on so-called "pan evaporation", the amount of water lost from a pan of water located at the appropriate site. However, pan evaporation made from a land-based site will usually grossly overestimate true evaporation because the pan gets warmer than the lake water and the air may be less humid over the pan than over the middle of the lake. You can float the pan in the lake, which is an improvement, but it is harder to monitor, water might spill out if it gets choppy, the circulation in the pan is not the same as in the lake ... you get the idea. Besides which, evaporation will vary widely with the wind , the time of day, the season of the year, and you will never have all the data you need, especially for a predictive model. Thus, for modeling purposes it is often necessary to estimate evaporation rates from empirical formulae that relate evaporation to the relevant meteorological conditions. A typical formula is

$$\phi_e = H_L K_e(\rho_{vs} - \rho_{va})$$

where

ρ_{vs} = saturated water vapor density at T_{surf}
ρ_{va} = water vap. dens. in the air at 10 m height
K_e = empirical bulk transport coeff. ($\approx 10^{-2}$–10^{-3} m/s)

HEAT FLUXES DUE TO CONDUCTION:

Sensible heat can be conducted from air to water or vice versa. The actual heat transfer may be due to molecular diffusion or turbulent diffusion. We will get into the particulars of that later, but we can state now that turbulent diffusion is usually several orders of magnitude greater than molecular diffusion. Unfortunately, turbulent heat transfer depends strongly on the structure of the turbulence in the air and in the water. We will deal with this is some greater detail later in the course, but even the most sophisticated models cannot fully predict turbulent transfers. We can use an empirical relationship that probably works modestly well if you have a good database for the lake which you can use to calibrate the model. A typical empirical formula for ϕ_c, the conductive heat flux is:

$$\phi_c = \rho_a C_{pa} K_c (T_s - T_a)$$

where:

ρ_a = density of air
C_{pa} = specific heat of air at const. pressure
K_c = empirical cond. heat transfer coeff. ($\approx 10^{-2}$–10^{-3} m/s)
T_s = surface temp. of the water
T_a = air temperature at 10 m height

It is <u>very</u> difficult to calibrate such a formula because there is no direct method for measuring conduction. The best you can do is accurately measure everything else and then deduce ϕ_c by difference.

Our overall heat balance equation is:

$$\phi_{net} = \phi_e + \phi_c + \phi_r$$

which can be expanded to:

$$\phi_{net} = \phi_e + \phi_c + \phi_s + \phi_l + \phi_l^*$$

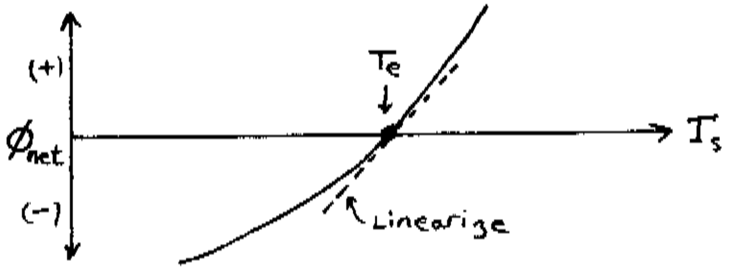
which we regroup as:

$$\phi_{net} = \underbrace{[\phi_e + \phi_c + \phi_l]}_{\substack{\uparrow \\ \text{These all have} \\ \text{some dependence} \\ \text{on } T_{surf} \\ \uparrow \\ \text{Hence these are} \\ \text{FEEDBACK COMPONENTS}}} + \underbrace{[\phi_s + \phi_l^*]}_{\substack{\uparrow \\ \text{These have a} \\ \text{strictly meteorological} \\ \text{or astronomical basis} \\ \uparrow \\ \text{These are external} \\ \text{to the system}}}$$

If the net influx of heat increases due to external forces (say an unusually sunny day), the water gets warmer, and as the water gets warmer, evaporation, conduction outward, and longwave backradiation all increase. These enhanced outward fluxes tend to counteract the influx and drive the net flux back towards a neutral value. For a given set of external (meteorological/astronomical) conditions:

$$\phi_{net}(T_s) = \phi_T(T_s) + \text{Constant}$$

which states that the net heat flux is a function only of surface temperature and that the net flux can be expressed by a sum of a composite of the surface-temperature-dependent "feedback" fluxes plus a constant that reflects the constant external conditions. Because the feedback is negative, the system tends to drive toward a neutral equilibrium state: for a given set of constant (on the average) external conditions, a body of water will attain its *equilibrium temperature*, T_e, at which point the net heat flux drops to zero.



If we linearize the net flux function in the vicinity of the equilibrium temperature as a first approximation, we can obtain the following:

$$\phi_{net} = k_e(T_s - T_e)$$

The value of k_e does not vary too much, and values in the range of 20–100 J m^{-2} s^{-1} $°C^{-1}$ are typical.