Given: \( P = 10 \), \( \phi = 20^\circ \)

15-tooth Pinion \( \Rightarrow \) \( d_p = 1.5 \)
25-tooth Gear \( \Rightarrow \) \( d_g = 2.5 \)

\( F_t = 30 \text{ lbs} \)

Find: \( D_x, D_y, D_z, D_r \)

\( C_x, C_y, C_z, C_r \)

Solution

\( F_t = 30 \text{ lbs} \)

\[ F_{r,p} = F_{a,g} = \frac{F_t \tan(\phi) \cos(\delta)}{2} \]

Using geometry:

\[ \tan \gamma = \frac{d_p}{d_g} = \frac{1.5}{2.5} = 0.6 \Rightarrow \gamma = 31^\circ \]

\& \( \Gamma = 59^\circ \)

Therefore:

\[ F_{a,g} = 30 \tan(20^\circ) \cos(31^\circ) = 9.35 \text{ lbs} \]

\( D_y \text{ Ans} \)

Also:

\[ F_{a,p} = F_{a,g} = \frac{F_t \tan(\phi) \sin(\delta)}{2} \]

\[ = 30 \tan(20^\circ) \sin(31^\circ) = 5.62 \text{ lbs} \]
\[
a = \frac{1}{2} F \cos \Gamma
\]

\[
= \frac{1}{2} \left( \frac{1}{2} \right) \cos (59) \tag{59}
\]

\[
a = 0.129
\]

\[
\frac{q}{16} + 0.129 = 0.691
\]

\[
\Sigma M_D = 0 \quad \text{About Z axis}
\]

\[
-C_x (.625) + 5.62 (.691) - 9.35 (b) = 0
\]

Where

\[
b = \frac{d_g}{z} - \frac{1}{2} F \sin \Gamma
\]

\[
= \frac{2.5}{2} - .5 (\sin 59) \sin (59) = 1.125
\]

\[
C_x = -10.6 \quad \text{lbs} \quad \text{ANS}
\]

\[
\Sigma M_o = 0 \quad \Rightarrow -C_x (.625 + .691) + D_x (.691) = 0
\]

\[
D_x = -4.98 \quad \text{lbs} \quad \text{ANS}
\]
Now \[ \sum M_{D_x} = 0 \] about \( x \)-axis

\[-C_z (0.625) + 30 (0.691) = 0\]

\[ \Rightarrow \ C_z = 33.2 \text{ lbs} \]

and \[ \sum F_z = 0 \]

\[ C_z + D_z + 30 = 0 \Rightarrow D_z = -63.2 \text{ lbs} \]

Radial forces

\[ D_R = \sqrt{D_x^2 + D_z^2} = \sqrt{20.2^2 + 63.2^2} = 66.3 \text{ lbs} \longrightarrow \text{ANS} \]

\[ C_R = \sqrt{C_x^2 + C_z^2} = \sqrt{10.6^2 + 33.2^2} = 34.8 \text{ lbs} \longrightarrow \text{ANS} \]
Given: Helical gear set

- \( \Phi_m = 20 \)
- \( \Psi = 30 \)
- Shaft b & idler \( F_t^2 = 500 \) lbs
- \( P_m = 7 \) teeth/min Gears 3 and 4
- \( N_3 = 54 \)
- \( N_4 = 14 \)

Find: Gear forces on shaft b

For Helical gears

\[ F_r^2 = F_t^2 \tan \Phi \]

Where \( \tan \Phi = \frac{\tan \Phi_m}{\sin \Psi} = \frac{\tan 20}{\sin 30} = 0.42 \)

\[ F_r^2 = 500 \times 0.42 = 210 \text{ lbs} \]

\[ F_a^2 = F_t^2 \tan \Psi = 500 \times \tan 30 = 289 \text{ lbs} \]

Diagram: Force vectors for gears 3 and 4.
Since \( b \) is an idler

\[ F_t \cdot d_4 = 500 \cdot d_3 \]

using geometric relations

\[ P = P_n \cos \varphi = 760 \cdot 30 = 6.06 \]

and

\[ P = \frac{N}{d} \Rightarrow d_3 = \frac{N_3}{P} = \frac{54}{6.06} = 8.91'' \]

and

\[ d_4 = \frac{14}{6.06} = 2.31'' \]

Therefore

\[ F_t^{(4)} (2.31) = 500 \cdot (8.91) \Rightarrow F_t^{(4)} = 1929 \text{ lbs} \]

For gear 4

\[ F_r^{(4)} = F_t^{(4)} \tan \varphi = 1929 \cdot (0.42) = 810 \text{ lbs} \]

and

\[ F_a^{(4)} = F_t^{(4)} \tan \psi = 1929 \cdot (\tan 30) = 114 \text{ lbs} \]

Since both gears are LH, their axial forces act

always in the same direction.
Given:

- RH Worm (LH gear)
- Single thread
- Power = 2000 W @ 600 rpm
- \( N_g = 48 \)
- \( P_a = 25 \text{ mm} \) (worm / gear)
- \( \phi_m = 14.5 \text{ mm} \)
- \( d_w = 100 \text{ mm} \)
- \( F_w = 100 \text{ mm} \)
- \( F_g = 50 \text{ mm} \)

Determine:
- Which bearing is thrust
- Reaction forces at A and B
Transmitted load

\[ \text{Power} = \text{Watts} = T \text{W} \]

\[ W = \frac{2 \pi n}{60} = \frac{2 \pi (600)}{60} = 62.8 \text{ rad/s} \]

Therefore, \[ 2000 = T \times (62.8) \Rightarrow T = 31.8 \text{ N.m} \]

using the torque formula

\[ T = F \cdot (\frac{d\omega}{dt}) \Rightarrow F = \frac{31.8 \times (2)}{0.1} = 636 \text{ N} \]

The worm forces are

\[ F_{wa} = \frac{(0 \cdot \Phi_m \cos \lambda - f \Delta \sin \lambda)}{(0 \cdot \Phi_m \Delta \sin \lambda + f \cos \lambda)} \]

the lead angle is

\[ \tan \lambda = \frac{1}{\pi d} = \frac{25}{\pi (100)} = 0.08 \Rightarrow \lambda = 4.55^\circ \]

Helix angle is the same \( \Psi = 4.55^\circ \)

To estimate the coefficient of friction, we estimate the sliding speed

\[ V_s = |V_w + V_g| \quad V_w = \text{relative pitch-line velocities} \]
Where pitch-line velocity of worm is

\[ V_w = |\vec{V}_w| = r \omega = \frac{d_w}{2} \omega_w \]

\[ V_w = \frac{100}{2} (62.8) = 3140 \text{ mm/sec} \]

and

\[ V_g = \frac{d_g}{2} \omega_g \]

The axial pitch of the worm must be equal to the circular pitch of gear on the plane of rotation

\[ P = 25 \text{ mm} \]

and

\[ P \rho = \pi \Rightarrow P = \frac{\pi}{25} = 0.125 \text{ teeth/mm or } m = 8 \]

also

\[ m = \frac{d}{N} \Rightarrow 8 = \frac{d_g}{48} \Rightarrow d_g = 384 \text{ mm} \]

The speed ratio is

\[ \frac{\omega_g}{\omega_w} = \frac{N_w}{N_g} = \frac{1}{48} \Rightarrow \omega_g = \frac{\omega_w}{48} = 1.3 \text{ rad/s} \]

Embed into \( V_g \) formula

\[ V_g = \frac{384}{2} (1.3) = 250 \text{ mm/s} \]
Combining $V_w$ and $V_g$, we get

\[ V_s = \frac{V_w}{\cos \lambda} \]

\[ V_s = \frac{3140}{\cos 4.55} = 3150 \text{ mm/s} \]

Converting to ft/min

\[ V_{si} = 3150 \left( \frac{1}{25.4} \right) \cdot \left( \frac{1}{12} \right) \cdot (60) = 620 \text{ ft/min} \]

\( f = 0.043 \) from table 13-42

Now

\[ F_{wa} = \frac{C_o (14.5) C_o (4.55) - 0.043 \sin (4.55)}{C_o (14.5) \sin (4.55) + 0.043 C_o (4.55)} \]

\[ F_{wa} = 5111 \text{ N} \]

and

\[ F_{wr} = \frac{\sin \phi_m}{C_o \phi_m \sin \lambda + f C_o \lambda} F_{wt} \]

\[ = \frac{\sin 14.5}{C_o 14.5 \sin 4.55 + 0.043 C_o 4.55} \]

\[ = \frac{636}{1330} = 0.476 \text{ N} \]
In a RH worms, the teeth recede CW from an observer looking down the axis of the worm when teeth turn in CW direction.

Therefore

When turning a RH worm CW, the teeth appear to be moving toward the observer. When turning a RH worm CCW, the teeth move away.

In this case, rotation is CCW (observer at z point), so teeth would move away pushing the gear, therefore the force direction is toward right (A is the thrust)
Given

\( \Phi = 20^\circ \) Spur Pinion

\( N_p = 16 \)

\( N_g = 48 \)

\( n_p = 300 \text{ rpm} \)

\( F = 2" \)

\( P = 6 \text{ teeth/m} \)

Grade-1 gears Through hardened @ 200 BHN

\( q_v = 6 \), un crowned

Accurate & Rigid Mounting

\( L_p = 10^8 \) cycles

\( R = 0.90 \)

Find Bending stress \((\text{AGMA})\)

Factor of safety for 5 hp Power transmission

Solution

\[ J = W K_o K_v K_s \frac{P_d}{F} \cdot \frac{K_m K_b}{J} \]
Where

\[ W = \frac{33000 \cdot \text{hp}}{V} = \frac{33000 \cdot (5)}{209.4} = 788 \text{ lbs} \]

\[ V = \frac{\pi \cdot \text{nd}}{12} = \frac{\pi \cdot (300) \cdot (2.667)}{12} = 209.4 \text{ ft/min} \]

\[ d = \frac{N_p}{P} = \frac{16}{6} = 2.667 \text{ inch} \]

\[ K_0 = 1 \] uniform loading

For \( Q_v = 6 \) and \( V = 209 \text{ ft/min} \) from Eq 14-28

\[ B = 0.25 \left( \frac{12-6}{3} \right) = 0.825 \]

\[ A = 50 + 56 \left( 1 - 0.825 \right) = 59.8 \]

\[ K_V = \left( \frac{59.8 + \sqrt{209.4}}{59.8} \right)^{0.825} = 1.196 \]

\[ K_S = 1 \text{ size factor} \]

\[ K_m = \text{load distribution factor} \quad K_m = 1 \]

\[ K_B = \text{Rim thickness factor} \quad K_B = 1 \]

\[ J = \text{Geometry factor for } N_p = 16 \text{ and } N_g = 48 \]

\[ J = 0.27 \]
Therefore

\[ \sigma = 788(1)(1.196)(1)(\frac{6}{2})(\frac{1}{0.27}) \]

\[ \sigma = 104.72 \text{ psi} \]

On the strength side

\[ \sigma_{\text{all}} = \frac{S_t}{S_F} \cdot \frac{Y_N}{K_T K_R} \quad \Rightarrow \quad S_f = \frac{28260 \cdot 0.977}{104.72 \cdot (1)(0.85)} = 3.14^* \]

Where from Fig. 14-2

\[ S_t = 77.3 (200) + 12800 = 28260 \]

\[ S_F = \text{To be found} \]

\[ Y_N = \text{stress cycle factor} \]

\[ Y_N = 1.6831 \cdot 10^8 - 0.0323 \]

\[ 0^* \]

\[ Y_N = 1.08558 \cdot 0.0178 = 0.977 \]

\[ K_T = 1 \]

\[ K_R = 0.85 \]

* Factor of safety against surface damage is about 1.
#14-24

\[ \Phi = 20^\circ \]

\[ N_p = 22 \]

\[ N_g = 60 \]

\[ P = 4 \text{ teeth/m} \]

\[ F = 3.25 \text{ in} \]

\[ N_p = 1145 \text{ rpm} \]

\[ L = 3 \times (10^9) \text{ cycles} \]

\[ Q_v = 6 \]

Material: 4340 through hardened grade 1 \( H_b = 250 \)

Loading: moderate shock

Power: smooth

\[ R = 0.99 \]

Find \( \psi_p \) (rating)

\[ \sigma = W K_o K_v K_s \frac{P_d}{F} \cdot \frac{K_m K_b}{J} \]

where \( W^{t} = \) unknown - to be determined
\[ K_0 = 1.25 \quad \text{from Fig 14-17} \]

\[ K'_v = \left( \frac{59.77 + \sqrt{1649}}{59.77} \right) \times 0.8255 = 1.534 \]

\[ d_p = \frac{22}{4} = 5.5 \quad \text{in} \]

\[ d_g = \frac{60}{4} = 15 \quad \text{in} \]

\[ V = \frac{\pi (5.5)(1145)}{12} = 1649 \quad \text{ft/min} \]

\[ B = 0.25 (12 - 6) = 0.8255 \]

\[ A = 50 + 56 (1 - 0.8255) = 59.77 \]

\[ K_S = 1 \]

\[ K_m = K_8 = 1 \]

\[ J = 0.34 \]

Therefore, \[ \sigma = W^t (1.25)(1.534)(1) \times \frac{4}{3.25} \times \frac{(1)(1)}{0.34} \]

\[ \sigma = 6.94 \quad \text{W}^t \]

On the Strength Side:

\[ \sigma_{all} = \frac{S_t}{S_F} \times \frac{Y_N}{K_T K_R} \]

\[ S_t = 37.3 (250) + 12800 = 32125 \quad \text{Psi} \]
\[ Y_N = \text{stress Cycle Factor (lives other than 10 million)} \]

\[ Y_N = 1.6831 \left( 3 \times 10^9 ight)^{-0.0323} = 0.861 \]

\[ K_T = 1 \]

\[ K_R = 1.00 \]

Therefore

\[ \sigma_{all} = \frac{32125}{1} \cdot \frac{0.861}{1} = 27685.5 \text{ psi} \]

Equating \( \sigma_{all} \) and \( \sigma \)

\[ 27685.5 = 6.94 W^t \Rightarrow W^t = 3989.3 \]

and

\[ h_p = \frac{W^t}{33000} = \frac{3989.3 \cdot 1649}{33000} = 199.3 \text{ hp}^* \]

\* Based on surface wear damage \( h_p = 53 \text{ hp} \)