Smoothing Analysis of Analog to Digital Converter Data

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Outline

- Objective and Significance
- Background of Data
- Experiment Methods
- Experiment Result
- Conclusion

Objective

- Find an optimal curve fit for the data of INL and loss of SNR
- Find an optimal curve fit for the data of mean DNL and loss of SNR

Significance

- Predict the SNR performance of a tested ADC according to its INL error
- Predict the improved SNR performance of tested ADCs according to mean DNL error
- Judge if the midpoint method can improve the ADC to required performance

Basic Concept

INL error

DNL error
Midpoint Correction Method 1.
- Spectrum of a sinusoid signal quantized by an 8-bit ADC with INL error

Midpoint Correction Method 2.
- Transfer function before correction

Midpoint Correction Method 3.
- Transfer function after correction
- Equivalent transfer function

Polynomial Smoothing

\[ y_i = w_0 + \sum_{j=1}^{J-1} w_j x_{i,j} \]

\[ w = (A^T A)^{-1} A^T \]
Leave One Out CVE of Polynomial Smoothing

Optimal Parameter: 6
Minimal CVE : 1.700236

Leave One Out CVE of Smoothing Spline

Optimal Parameter: 0.102414
Minimal CVE: 1.754758

Leave One Out CVE of Kernel Smoothing

Optimal Parameter: 0.526207
Minimal CVE : 2.188440

Smoothing Spline

\[ E_\alpha = \alpha \sum_{i=1}^{n} (y_i - \hat{g}(x_i))^2 + (1-\alpha) \int (\hat{g}(x))^2 \, dx \]

\( \alpha : \) Parameter controls the tradeoff of fitness and smoothness

\( \alpha = 0 : \) Least square straight line

\( \alpha = 1 : \) Interpolating cubic spline

Kernel Smoothing

\[ E[y|x|g(x)] = \frac{\sum_{i=1}^{n} h_i |x-x_i| g(x_i)}{\sum_{i=1}^{n} h_i |x-x_i|} = \int \frac{h(x-x_i)}{\int h(x-x_i)} \]

Weighted Averaging

\[ g(x) = \frac{\sum_{i=1}^{n} b_i (|x-x_i|) y_i}{\sum_{i=1}^{n} b_i |x-x_i|} = \frac{\sum_{i=1}^{n} b_i y_i}{\sum_{i=1}^{n} b_i} \]

Biweight function: \( b_i = (1 - \frac{d_i}{d_{max}}) \)

\( d_{max} \)
Leave One Out CVE of Weighted Averaging

Result Comparison (Data 1)

<table>
<thead>
<tr>
<th>Method</th>
<th>Minimal CVE</th>
<th>Optimal Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomial</td>
<td>1.700236</td>
<td>6</td>
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<tr>
<td>Smoothing Spline</td>
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<td>Kernel Smoothing</td>
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<td>Weighted Local Averaging</td>
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Optimal Smoothing 1 (Data 1)

Optimal Smoothing 2 (Data 1)

Optimal Smoothing 3 (Data 1)

Optimal Smoothing 4 (Data 1)
Conclusion 1

- For data 1, the optimal model is polynomial smoothing and the optimal order is 6

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<tr>
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<th>Optimal Parameter</th>
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Optimal Smoothing 1 (Data 2)

Optimal Smoothing 2 (Data 2)

Optimal Smoothing 3 (Data 2)

Optimal Smoothing 4 (Data 2)
Conclusion 2

For data 2, the optimal model is polynomial smoothing and the optimal order is 6