Time-Series Prediction

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Outline
- Introduction
- Methods
  - Linear Model
  - Non-Linear Model
- Results
- Conclusions

Time Series Prediction
- Predicting the future based on collected data
- Different Areas of Application
  - Scientific
    - Electrical load
  - Economic
    - Stock Market
    - Currency Exchange

IJCNN Prediction Competition
- Predict the output of two time series generated from the same algorithm

The data was given with non-constant time interval.
- Used cubic spline to reevaluate data values at constant time interval.

Linear Model
- Model given as:
  - To find first predicted output
    - \[ \text{To find second predicted output} \]
    - \[ \text{To find predicted output} \]
    - \[ \text{To find second predicted output} \]
Set up A matrix for time series
- A and y matrix for linear model are given as:

\[
A = \begin{bmatrix}
1 & x_1 & x_2 & \cdots & x_n \\
1 & x_1 & x_2 & \cdots & x_n \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_1 & x_2 & \cdots & x_n \\
\end{bmatrix}, \quad y = \begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n \\
\end{bmatrix}
\]

- Finding the coefficients for the least squares approximation
  \[\omega = (A^T A)^{-1} A^T y\]

Calculate the predicted output
\[\hat{y}_t = \omega_0 + \sum_{j=1}^{p} \omega_j x_{ij}\]
Repeat for the specified number of predicted output values needed.

Non-Linear Model
- Multivariate Kernel Smoothing
  \[
  \hat{y}(x) = \sum_{i} \omega_i k(x_i, x)
  \]
- \(\phi = \sum_{i} (x_i - x)^T\) is the Euclidean distance between the segment predicting the output and the other segments in the series that give each output value.
- \(\sigma = \text{width of the bump}\)
- Used gaussian kernel
  - is given as: \[k(x) = e^{-\|x\|^2}\]

Results
- Compared last 500 points of given time-series versus the predicted 500 points.
- Found ASE value given by
  \[ASE = \frac{1}{n} \sum (\hat{y} - y)^2\]
- For each method, used lowest ASE value to predict the next 500 outputs to the given chaotic time series.

Plot for linear model with ASE values
Plot and ASE values for non-linear model with 500 inputs

Plot and ASE values for non-linear model with 150 inputs

Best Predicted Output

<table>
<thead>
<tr>
<th>Data set</th>
<th>ASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.949</td>
</tr>
<tr>
<td>Gaussian 150</td>
<td>0.809</td>
</tr>
<tr>
<td>Gaussian 500</td>
<td>0.791</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Data set</th>
<th>ASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.565</td>
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<tr>
<td>Gaussian 150</td>
<td>0.556</td>
</tr>
<tr>
<td>Gaussian 500</td>
<td>0.659</td>
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</table>
Coefficients for Linear Model

Conclusions

- Tough to make any assumptions from my studies.
- When taking every point, the kernel method had a better prediction for short periods of time.
  - Did not follow the real output for very long.
- Most that had low ASE values quickly became constant valued
- Ways to improve predictions
  - Taking more simulations with more refined widths.
  - Other types of models such as Neural Networks may give better predictions.