Overview of Communication Topics

- Sinusoidal amplitude modulation
- Amplitude demodulation (synchronous and asynchronous)
- Double- and single-sideband AM modulation
- Pulse-amplitude modulation
- Pulse code modulation
- Frequency-division multiplexing
- Time-division multiplexing
- Narrowband frequency modulation

Handy Trigonometry Identities

\[
\begin{align*}
\cos(a + b) &= \cos(a) \cos(b) - \sin(a) \sin(b) \\
\sin(a + b) &= \sin(a) \cos(b) + \cos(a) \sin(b) \\
\cos(a) \cos(b) &= \frac{1}{2} \cos(a - b) + \frac{1}{2} \cos(a + b) \\
\sin(a) \sin(b) &= \frac{1}{2} \cos(a - b) - \frac{1}{2} \cos(a + b) \\
\sin(a) \cos(b) &= \frac{1}{2} \sin(a - b) + \frac{1}{2} \sin(a + b)
\end{align*}
\]

Introduction to Communication Systems

- Communications is a very active and large area of electrical engineering
- Experienced a lot of growth through the nineties with the advent of wireless cell phones and the internet
- Still an active area of research
- Fundamentals of signals and systems are essential to grasping communications concepts
- The next two lectures will merely introduce some of the fundamental concepts
- Will primarily focus on modulation and demodulation in continuous-time
- Analogous concepts apply in discrete-time

Introduction to Amplitude Modulation

\[
x(t) \rightarrow x(t) \times c(t) \rightarrow y(t)
\]

\[
y(t) = x(t) \cdot c(t) \quad c(t) = \cos(\omega_c t + \theta_c)
\]

- **Modulation**: the process of embedding an information-bearing signal into a second signal
- **Demodulation**: extracting the information-bearing signal from the second signal
- **Sinusoidal Amplitude modulation**: a sinusoidal carrier \(c(t)\) has its amplitude modified by the information-bearing signal \(x(t)\)
Fourier Analysis of Sinusoidal Amplitude Modulation

For convenience, we will assume $\theta_c = 0$.

\[
\begin{align*}
  c(t) &= \cos(\omega_c t) \\
  C(j\omega) &= \pi \left[ \delta(\omega - \omega_c) + \delta(\omega + \omega_c) \right] \\
  y(t) &= x(t) \cdot c(t) \\
  Y(j\omega) &= \frac{1}{2\pi} \left[ X(j\omega) \ast C(j\omega) \right] \\
  X(j\omega) \ast \delta(\omega - \omega_c) &= X(j(\omega - \omega_c)) \\
  Y(j\omega) &= \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))
\end{align*}
\]

Thus, sinusoidal AM shifts the baseband signal $x(t)$ so that it is centered at $\pm \omega_c$.

Thus, $x(t)$ can be recovered only if $\omega_c > \omega_x$ so that the replicated spectra don’t overlap.

Example 1: Sinusoidal AM of a Random Signal

```
Example of Sinusoidal Amplitude Modulation

Example of Sinusoidal Amplitude Modulation
```

Example 1: MATLAB Code

```
function [] = AMTimeDomain();

close all;

N = 2000; % No. samples
fc = 50e3; % Carrier frequency
fs = 1e6; % Sample rate

k = 1:N;
t = (k-1)/fs;

xh = randn(1,N); % Random high-frequency signal
[n,wn] = ellipord(0.01,0.02,0.5,60);
[b,a] = ellip(n,0.5,60,wn);
x = filter(b,a,xh); % Lowpass filter to create baseband signal

c = cos(2*pi*fc*t);
y = x.*c;

figure;
FigureSet(1,'LTX');

subplot(3,1,1);
h = plot(t,x,'b');set(h,'LineWidth',0.2);xlim([0 max(t)]);ylim([-0.3 0.3]);ylabel('x(t)');
title('Example of Sinusoidal Amplitude Modulation');
box off;AxisLines;

subplot(3,1,2);
h = plot(t,c,'b');set(h,'LineWidth',0.2);xlim([0 max(t)]);ylim([-1.1 1.1]);
```

```
Synchronous Sinusoidal Amplitude Demodulation

\[
\begin{align*}
\text{Transmitter} & \quad y(t) = x(t) \cos(\omega_c t) \\
\text{Channel} & \quad c(t) = y(t) \\
\text{Receiver} & \quad H(s) = w(t) \\
\text{Demodulator} & \quad \hat{x}(t) = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos(2\omega_c t)
\end{align*}
\]

- **Synchronous** demodulation assumptions
  - The carrier \( c(t) \) is known exactly
  - \( \omega_c > \omega_x \)
- The \( x(t) \) can be extracted by multiplying \( y(t) \) by the **same carrier** and lowpass filtering the signal

Fourier Analysis of Sinusoidal AM Demodulation

\[
\begin{align*}
\text{Transmitter} & \quad \hat{x}(t) \\
\text{Channel} & \quad y(t) = x(t) \cos(\omega_c t) \\
\text{Receiver} & \quad H(s) = y(t) \\
\text{Demodulator} & \quad \hat{x}(t) = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos(2\omega_c t)
\end{align*}
\]

- The lowpass filter \( H(s) \) should have a passband gain of 2
- The transition band is very wide so the filter does not need to be close to ideal (e.g. it can be low order)
- We learned how to design this type of filter in ECE 222
- We assumed the signal spectrum \( X(j\omega) \) was real
- The same ideas hold if \( X(j\omega) \) is complex
- Called synchronous demodulation because we assumed the transmitter and receiver carrier signals \( c(t) \) were in phase
**Introduction to Asynchronous AM Demodulation**

- **Asynchronous modulation** does not require the carrier signal $c(t)$ be available in the receiver.
- Thus, there is no need for synchronization.
- Asynchronous Modulation Assumptions:
  
  $\omega_c \gg \omega_x$
  
  $x(t) > 0$ for all $t$

- However, it does require that the baseband signal $x(t)$ be positive.
- In this case, the envelope of the modulated signal $y(t)$ is approximately the same as the baseband signal $x(t)$.
- Thus, we can recover a good approximation of $x(t)$ with an envelope detector.

**Example 2: Asynchronous Amplitude Modulation**

- **Example of Asynchronous Sinusoidal AM Modulation**

  $w(t) = \frac{1}{2} x(t) \cos(\theta - \phi) + \frac{1}{2} x(t) \cos(2\omega_c t + \theta + \phi)$

  - If $\theta = \phi$ then we have the same case as before and we recover $x(t)$ exactly after a lowpass filter with a passband gain of 2.
  - If $|\theta - \phi| = \frac{\pi}{2}$, we lose the signal completely.
  - Otherwise, the received signal is attenuated.
  - The phase relationship of the oscillators must be maintained over time.
  - This type of careful synchronization is difficult to maintain.
  - Phase-locked loops (PLL) can be used to solve this problem.
  - In ECE 323 you will design and build PLL’s.
  - The carrier frequency $\omega_c$ of the transmitter and receiver must also be the same and remain so over time.
Diodes can be used as a key component in envelope detectors.

- Like resistors, diodes do not have memory and the relationship between voltage and current does not depend on time.
- Key idea: diodes only allow current to flow in one direction.
- When they are on \((V \geq 0.7\) V\), they act like an ideal voltage source.
- When they are off \((V < 0.7\) V\), they act like an open circuit.

Example 2: MATLAB Code

```matlab
function [] = AAMTimeDomain();
close all;
N = 2000; % No. samples
fc = 50e3; % Carrier frequency
fs = 1e6; % Sample rate
k = 1:N;
t = (k-1)/fs;
xh = rand(1,N)-0.5; % Random high-frequency signal limited to [-0.5 0.5]
[b,a] = ellipord(0.02,0.03,0.5,60);
x = filter(b,a,xh); % Lowpass filter to create baseband signal
x = x + 0.2; % Convert to positive signal
c = cos(2*pi*fc*t);
y = x.*c;
figure;
FigureSet(1,'LTX');subplot(3,1,1);
plot(t,x,'b');set(h,'LineWidth',0.2);xlim([0 max(t)]);ylim([-0.1 0.4]);
ylabel('x(t)');title('Example of Asynchronous Sinusoidal AM Modulation');box off;AxisLines;
subplot(3,1,2);
plot(t,c,'r');set(h,'LineWidth',0.2);xlim([0 max(t)]);
ylabel('c(t)');box off;AxisLines;
subplot(3,1,3);
plot(t,y,'g',t,x,'b',t,-x,'b');set(h,'LineWidth',0.2);
xlim([0 max(t)]);ylim([-0.39 0.39]);xlabel('Time (s)');ylabel('y(t)');box off;
AxisLines;
AxisSet(8);print -depsc AAMTimeDomain;
```

Example 3: Full Wave Rectifier

Draw the equivalent circuits for \(v_s > 0\) and \(v_s < 0\) assuming an ideal model of the diode with a threshold voltage of 0 V.
Example 4: Envelope Tracking

1. Example of Envelope Tracking of an Asynchronous AM Signal

2. A diode can be used to extract the upper half of the modulated signal

3. This is called a half-wave rectifier

4. Roughly speaking, when \( y(t) > 0 \), \( e(t) \approx y(t) \)

5. By connecting a capacitor in parallel with the resistor, the \( RC \) circuit acts like a first-order lowpass filter

6. This smooths the received waveform

7. A full-wave rectifier that recovers both the negative and positive peaks has better performance

MATLAB Code

```matlab
function [] = EnvelopeTracking();
    close all;
    N = 2000; % No. samples
    fc = 50e3; % Carrier frequency
    fs = 1e6; % Sample rate
    al = 0.95; % First-order filter parameter
    k = 1:N;
    t = (k-1)/fs;
    rand('state',10);
    xh = rand(1,N)-0.5; % Random high-frequency signal limited to [-0.5 0.5]
    [n,wn] = ellipord(0.01,0.02,0.5,60);
    [b,a] = ellip(n,0.5,60,wn);
    x = filter(b,a,xh); % Lowpass filter to create baseband signal
    x = x + 0.2; % Convert to positive signal
    c = cos(2*pi*fc*t);
    y = x.*c;
    eh = y.*(y>0);
    rh = filter(1-al,[1 -al],eh-mean(eh))+mean(eh);
    ef = abs(y);
    rf = filter(1-al,[1 -al],ef-mean(ef))+mean(ef);

    figure;
    FigureSet(1,'LTX');
    subplot(3,1,1);
    h = plot(t,y,'g',t,x,'b',t,-x,'b');
    set(h,'LineWidth',0.2);
    xlim(1e-3*[0.8 1.8]);
    ylim([-0.39 0.39]);
    ylabel('y(t)');
    title('Example of Envelop Tracking of an Asynchronous AM Signal');
    box off;
```
Asynchronous Amplitude Modulation

\[ y(t) = X(t) + A \]

- Most baseband signals will not be positive
- We can make amplitude-limited signals, \(|x(t)| \leq x_{\text{max}}\), positive by adding a constant \(A\) such that \(A > x_{\text{max}}\)
- The envelope detector then approximates \(x(t) + A\)
- \(x(t)\) can then be extracted with a highpass filter to remove \(A\) (the DC component)
- The ratio \(m = x_{\text{max}}/A\) is called the **modulation index**
- If expressed in percentage, \(100x_{\text{max}}/A\), it is called the **percent modulation**
- The spectrum of \(y(t)\) contains impulses to account for \(A\)

Asynchronous Amplitude Modulation Tradeoffs

- In most applications, the FCC limits the transmission power
- For asynchronous AM, transmitting the carrier component requires a portion of this power
- Thus, asynchronous AM is less efficient than synchronous AM
- However, the receiver is easier and cheaper to build
- As \(m \to 1\), more of the transmitter power is used for the baseband signal \(x(t)\)
- As \(m \to 0\), the signal is easier to demodulate with an envelope detector
Frequency-Division Multiplexing

- We can transmit multiple signals using a single transmitting antenna with frequency-division multiplexing (FDM)
- Each baseband signal is shifted to a different frequency band
- Thus, multiple baseband signals can be transmitted simultaneously over a single wideband channel
- The different modulated signals \( y_1(t), y_2(t), \) and \( y_3(t) \) are simply summed before sending to the antenna
- To recover a specific signal, the corresponding frequency band usually is extracted with a bandpass filter

Time-Division Multiplexing

- The sampling theorem tells us we can represent any bandlimited signals by its samples \( x(nT) \) as long as \( \omega_s > 2\omega_x \)
- Thus we can convert multiple bandlimited signals into discrete-time signals: \( x_1(t) \rightarrow x_1[n], x_2(t) \rightarrow x_2[n], x_3(t) \rightarrow x_3[n] \)
- Time-Division multiplexing interleaves these signals to form a composite signal
  \[ \ldots x_1[0], x_2[0], x_3[0], x_1[1], x_2[1], x_3[1], x_1[2], x_2[2], x_3[2] \ldots \]
- A different time interval is assigned to each signal
- We could then form a continuous-time signal using bandlimited interpolation
- If \( M \) signals are multiplexed and each signal has a bandwidth of \( \omega_x \), the multiplexed signal \( y(t) \) will require a bandwidth of \( M \times \omega_x \)

Single Sideband AM

- Let us define the bandwidth of the signal as \( \omega_x \), the highest frequency component of the signal
- The signal transmitted requires twice the bandwidth, \( 2\omega_x \)
- Near \( \omega_c \), the signal content for both negative and positive frequencies is transmitted
- We don’t need this much information to reconstruct \( X(j\omega) \)
- If we know \( X(j\omega) \) for either positive or negative frequencies, we can use symmetry to construct the other part
- Thus, we only need to transmit one of the sidebands

Single Sideband AM Continued

- What we have discussed so far uses double-sideband modulation
- We can use single-sideband modulation by removing the upper or lower sidebands
- Requires only half the bandwidth!
- An obvious approach: lowpass (to retain lower sidebands) or highpass filter (to retain upper sidebands)
- Requires a nearly ideal high-frequency filter
- SSB modulation increases the cost of the transmitter
- If asynchronous modulation is used, it also increases the cost and complexity of the receiver
Example 5: Time-Division Multiplexing

Example 5: MATLAB Code

```matlab
function [] = TDMultiplexing();
close all;
N = 10; % No. samples
k = 1:N;
x1 = rand(N,1);
x2 = rand(N,1);
x3 = rand(N,1);
y = zeros(3*N,1);
a1 = 1.5:3; n1; a2 = 2:3:3*n1; a3 = 3:3:3*n1;
y(k1) = x1;
y(k2) = x2;
y(k3) = x3;
ws = pi; T = 1; % Sample rate
m = 0:0.01:3*N; t = m; y = zeros(size(t)); % Reconstructed signal
yr = zeros(size(t)); % Reconstructed signal
for cnt = 1:length(m),
    yr = yr + (ws*T/pi)*y(cnt)*sinc(ws*(t-m(cnt)+7/3)/pi);
end;
figure;
axis([0 3*N+1]; ylim([-0.3 1.3]); ylabel('y[n]');
box off;
AxisLines;
AxisSet(6);
print -depsc TDMultiplexing;
```
Example 6: PCM

Create a random digital (discrete-time and discrete-valued) signal consisting of fifty 0’s and 1’s. Encode the baseband signal \( x(t) \) such that the bandwidth is limited to 100 Hz. What is the minimum time required to transmit the signal? Plot the discrete-time signal \( x[n] \), the baseband encoded signal \( x(t) \), and an "eye" diagram of the overlapping received pulses. Assume that the channel does not cause any distortion and that the receiver and transmitter sampling times are synchronized. Hint: recall that

\[
\frac{W}{\pi} \text{sinc} \left( \frac{tW}{\pi} \right) \Leftrightarrow P_W(j\omega)
\]
Example 6: MATLAB Code

```matlab
function [] = PCMEx();
close all;
N = 50; % No. samples
n = 1:N; % Discrete-time index
d = (rand(N,1)>0.5); % Digital signal
wc = 2*pi*50; % Limit pulse bandwidth to 100 Hz (-50 to 50)
T = pi/wc; % Sample period
Ts = 0.0005;
t = 0:Ts:(N+1)*T;
figure;
FigureSet(1,'LTX');subplot(2,1,1);
h = stem(n,d,'b');set(h(1),'MarkerSize',2);
set(h(1),'MarkerFaceColor','b');hold off;xlim([0 11]);ylim([0 1.05]);ylabel('x_1[n]');title('Example of Pulse-Code Modulation');box off;
subplot(2,1,2);
xc = zeros(size(t)); % Modulated signal x(t)
for cnt = 1:length(n),
s = -1*(d(cnt)==0) + 1*(d(cnt)==1);p = s*sinc(wc*(t-n(cnt)*T)/pi);
plot(t,p,'b');hold on;xc = xc + p;end;
plot(t,xc,'g');plot(n*T,-1*(d==0)+1*(d==1),'ro','MarkerSize',2,'MarkerFaceColor','r');hold off;
xlim([0 11*T]);ylabel('x(t)');box off;
AxisLines;AxisSet(6);print -depsc PCMSignals;
figure;
FigureSet(2,'LTX');for cnt = 1:length(n),
k = -round(T/Ts):round(T/Ts);
plot(k*Ts,xc(round(n(cnt)*T/Ts)+k+1));hold on;
end;hold off;
xlim([min(k*Ts) max(k*Ts)]);ylabel('x(t)');xlabel('Time (sec)');title('Eye Diagram');box off;
AxisSet(6);AxisLines;print -depsc PCMEyeDiagram;
```

Example 6: Plot of $x[n]$ and $x(t)$

Example of Pulse-Code Modulation

Example 6: Eye Diagram

Eye Diagram
Example 6: MATLAB Code

```matlab
function [] = PCMEx();
close all;

N = 50;   % No. samples
n = 1:N;  % Discrete-time index
x = (rand(N,1)>0.5); % Digital signal

w = 2*pi*50;  % Limit pulse bandwidth to 100 Hz (-50 to 50)
T = pi/w;     % Sample period
Ts = 0.0005;
t = 0:Ts:(N+1)*T;

figure;
FigureSet(1,'LTX');subplot(2,1,1);
h = stem(n,x,'b');set(h(1),'MarkerSize',2);
set(h(1),'MarkerFaceColor','b');hold off;xlim([0 11]);ylim([0 1.05]);ylabel('x_1[n]');title('Example of Pulse-Code Modulation');box off;
subplot(2,1,2);
x = zeros(size(t)); % Modulated signal x(t)
for cnt = 1:length(n),
s = -1*(x(cnt)==0) + 1*(x(cnt)==1);p = s*sinc(wc*(t-n(cnt)*T)/pi);
plot(t,p,'b');hold on;xc = xc + p;end;
plot(t,xc,'g');
plot(n*T,-1*(x==0)+1*(x==1),'ro','MarkerSize',2,'MarkerFaceColor','r');hold off;xlim([0 11*T]);ylabel('x(t)');box off;
AxisLines;
AxisSet(6);
print -depsc PCMSignals;
figure;
FigureSet(2,'LTX');for cnt = 1:length(n),
k = round(T/Ts):round(T/Ts);
plot(k*Ts,xc(round(n(cnt)*T/Ts)+k+1));hold on;
end;hold off;xlim([min(k*Ts) max(k*Ts)]);ylabel('x(t)');xlabel('Time (sec)');title('Eye Diagram');box off;
AxisLines;
AxisSet(6);
print -depsc PCMEyeDiagram;
```

Example 7: Noise Tolerance of PCM

Repeat the previous example, but this time assume that the channel adds noise that is uniformly distributed between -0.5 and 0.5. Can you still accurately receive the signal?
Example 7: MATLAB Code

```matlab
function [] = PCMNoisEx();

close all;

N = 50; % No. samples
n = 1:N; % Discrete-time index
xd = (rand(N,1)>0.5); % Digital signal
wc = 2*pi*50; % Limit pulse bandwidth to 100 Hz (-50 to 50)
T = pi/wc; % Sample period
Ts = 0.0002;
t = 0:T:Tstime;
x = xn(Ts);

figure;
FigureSet(1,'LTX');
subplot(3,1,1);
h = stem(n,xd,'b');
set(h(1),'MarkerSize',2);
set(h(1),'MarkerFaceColor','b');hold off;
xlim([0 11]);
ylim([0 1.05]);
ylabel('x_1[n]');
title('Example of Pulse-Code Modulation');
box off;

subplot(3,1,2);
xc = zeros(size(t)); % Modulated signal x(t)
for cnt = 1:length(n),
s = -1*(xd(cnt)==0) + 1*(xd(cnt)==1);
p = s*sinc(wc*(t-n(cnt)*T)/pi);
plot(t,p,'b');hold on;
xc = xc + p;
end;

subplot(3,1,3);
r = xc + (rand(size(xc))-0.5); % Add noise to the received signal
plot(t,r,'b');hold on;
plot(t,xc,'g');hold off;
xlim([0 11*T]);
ylabel('r(t)');
title('Eye Diagram');
box off;

AxisSet(6);
print -depsc PCMNoiseSignals;

depc -depsc PCMNoiseSignals;
end;
```

Example 8: Communication System

Design a system (transmitter and receiver) that transmits a stereo audio signal through a channel in the frequency band of 1.2 MHz ±40 kHz. Discuss the design tradeoffs of different approaches to this problem and sketch the spectrum of signals at each stage of the process.
Sinusoidal Angle Modulation

\[ c(t) = A \cos (\omega_c t + \theta_c) \quad \text{and} \quad c(t) = A \cos (\theta(t)) \]

- So far we have discussed different types of amplitude modulation
- **Angle Modulation** alters the angle of the carrier signal rather than the amplitude
- Define the instantaneous angle of the carrier signal \( c(t) \) as \( \theta(t) \)
- There are two forms of **angle modulation**
  - Phase modulation (PM): \( \theta(t) = \omega_c t + \theta_0 + k_p x(t) \)
  - Frequency modulation (FM): \( \frac{d\theta(t)}{dt} = \omega_c + k_f x(t) \)
- Note that for FM, \( \theta(t) \neq (\omega_c + k_f x(t)) t \)

Angle Modulation Versus Amplitude Modulation

Angle Modulation (say FM) versus Amplitude Modulation (AM)

+ One advantage of FM is that the amplitude of the signal transmitted can always be at maximum power
+ FM is also less sensitive to many common types of noise than AM
- However, FM generally requires greater bandwidth than AM
Example 9: Angle Modulation

Create a random signal bandlimited to ±1 Hz and amplitude limited to one (e.g. \(|x(t)| \leq 1\)). Modulate the signal use PM and FM with a carrier frequency of 3 Hz. Use \(k_p = 3\) and \(k_f = 4\pi\). Plot the baseband signal, the carrier signal, and the modulated signals.

Example 9: MATLAB Code

```matlab
function [] = AngleModulation();
close all;
N = 500; % No. samples
fc = 3; % Carrier frequency (Hz)
fs = 50; % Sample rate (Hz)
fx = 1; % Bandlimit of baseband signal
kp = 3; % PM scaling coefficient
kf = 2*pi*2; % FM scaling coefficient
wc = 2*pi*fc;k = 1:N;
t = (k-1)/fs;
xh = randn(1,N); % Random high-frequency signal
[n,wn] = ellipord(0.95*fx/(fs/2),fx/(fs/2),0.5,60);
[b,a] = ellip(n,0.5,60,wn);
x = filter(b,a,xh); % Lowpass filter to create baseband signal
x = x/max(abs(x)); % Scale so maximum amplitude is 1
c = cos(wc*t); % Carrier signal
yp = cos(wc*t + kp*x); % PM
theta = cumsum(wc + kf*x)/fs; % Approximate integral of angle
yf = cos(theta);
figure;
figureSet(1,'LTX');subplot(4,1,1);
h = plot(t,x,'b');set(h,'LineWidth',0.2);xlim([0 max(t)]);ylabel('x(t)');title('Example of Sinusoidal AM Modulation');
box off;AxisLines;subplot(4,1,2);
h = plot(t,c,'b');set(h,'LineWidth',0.2);xlim([0 max(t)]);ylabel('c(t)');box off;AxisLines;subplot(4,1,3);
h = plot(t,yp,'b');set(h,'LineWidth',0.2);xlim([0 max(t)]);xlabel('Time (s)');ylabel('PM');box off;AxisLines;subplot(4,1,4);
h = plot(t,yf,'b');set(h,'LineWidth',0.2);xlim([0 max(t)]);xlabel('Time (s)');ylabel('FM');box off; AxisSet(6);print -depsc AngleModulation;
```

Example 9: Signal Plot

![Signal Plot](image)
Carrier frequency is present and strong
• Like double-sideband AM

Bandwidth is twice that of $x(t)$
• Unlike AM, sidebands are out of phase by $180^\circ$

The instantaneous frequency varies between $\omega_c + kfA$ and $\omega_c - kfA$
• The modulated signal is then of the form

$$y(t) = \cos \left( \omega_c t + \int_0^t x(\tau) \, d\tau \right) = \cos \left( \omega_c t + \frac{\Delta \omega}{\omega_x} \sin(\omega_x t) + \theta_0 \right)$$
• Define the following variables

$$\Delta \omega \equiv kfA \quad m \equiv \frac{\Delta \omega}{\omega_x}$$
• $m$ is called the modulation index

**Narrowband Frequency Modulation Continued**

- Like AM, spectrum contains sidebands
- Unlike AM, sidebands are out of phase by $180^\circ$
- Bandwidth is twice that of $x(t)$ like double-sideband AM
- Carrier frequency is present and strong
Example 10: Angle Modulation

Create a sinusoidal baseband signal with a fundamental frequency of 1 Hz and a carrier sinusoidal signal at 12 Hz. Plot these signals and the modulated signals after applying amplitude modulation and frequency modulation. Use a scaling factor $k_f = 1$ and a modulation index of $m = 0.5$. Solve for the baseband signal amplitude $A$. 

Example 10: Relevant MATLAB Code

```matlab
%function [] = AMFM();
close all;
fx = 1; % Signal frequency (Hz)
fC = 15; % Carrier frequency (Hz)
fs = 100; % Sampling frequency
kf = 1; % FM scaling coefficient
m = 0.5; % Modulation index
wx = 2*pi*fx; % Signal frequency (rad/s)
wC = 2*pi*fC; % Carrier frequency (rad/s)
A = m*wx/kf; % Modulating amplitude

x = A*cos(wx*t); % Baseband signal
c = cos(wC*t); % Carrier signal
ya = x.*c; % Amplitude modulated signal
yf = cos(wC*t + m*sin(wx*t)); % Frequency modulated signal

figure;
FigureSet(1,'LTX'); subplot(4,1,1);
h = plot(t,x,'b');set(h,'LineWidth',0.2);xlim([min(t) max(t)]);
ylabel('x(t)'); title('Example of Sinusoidal AM and FM Modulation'); box off;AxisLines;

subplot(4,1,2);
h = plot(t,c,'b');set(h,'LineWidth',0.2);xlim([min(t) max(t)]);
ylabel('c(t)');

subplot(4,1,3);
h = plot(t,ya,'b',t,x,'r',t,-x,'g');set(h,'LineWidth',0.2);xlim([min(t) max(t)]);
xlabel('Time (s)'); ylabel('AM'); box off;AxisLines;

subplot(4,1,4);
h = plot(t,yf,'b');set(h,'LineWidth',0.2);xlim([min(t) max(t)]);
xlabel('Time (s)'); ylabel('FM'); box off;AxisLines;
AxisSet(6);
print -depsc AMFM;
```

Example 10: Signal Plot

![Example of Sinusoidal AM and FM Modulation](image)
Summary

- Modulation is the process of embedding one signal in another with desirable properties for communication
- Most forms of modulation are nonlinear
- Sinusoidal AM is relatively simple and inexpensive
- Synchronous AM is more efficient than asynchronous AM, but is also more expensive
- FM is more tolerant of noise than FM, but requires more bandwidth and cost
- Filters and frequency analysis using the Fourier transform have a crucial role in communication systems
- Frequency- (FDM) and time-division (TDM) multiplexing can be used to merge multiple bandlimited signals into a single composite signal with a larger bandwidth