Exam 2 Solutions
May 9, 2005

ECE 223: Signals & Systems II
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- Write your name above.
- Keep your exam flat during the entire exam.
- If you have to leave the exam temporarily, close the exam and leave it face down while you are out of the room.
- Turn off any cell phones or pagers that might interrupt the exam.
- Do not open the exam until instructed to do so.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- You will have 100 minutes to complete the exam.
- If you have extra time, double check your answers.
- Remember to include units with each of your answers.

Problem 1:______ / 14
Problem 2:______ / 10
Problem 3:______ / 13
Problem 4:______ / 13

Total:______ / 50
1. **Transform Properties (14 pts)**

The following abbreviations are used throughout this exam: FT = Fourier Transform, FS = Fourier Series, DT = Discrete-Time, and CT = Continuous-Time. Assume the signal is real-valued for all of these questions.

a. (1 pt) Suppose the real part of a transform is set to 0. What part of the signal is recovered when the synthesis equation is applied to only the imaginary part of the transform?
   
   Only the odd part is recovered.

b. (1 pt) Which analysis equations are only applied to a segment of the signal?

   \[
   \text{DTFT} \quad \text{DTFS} \quad \text{CTFT} \quad \text{CTFS}
   \]

c. (1 pt) Which synthesis equations are only applied to a segment of the transform?

   \[
   \text{DTFT} \quad \text{DTFS} \quad \text{CTFT} \quad \text{CTFS}
   \]

d. (1 pt) Which transforms are best suited for the analysis of periodic signals?

   \[
   \text{DTFT} \quad \text{DTFS} \quad \text{CTFT} \quad \text{CTFS}
   \]

e. (1 pt) Which transform synthesis equations can be written as weighted sums or integrals of sine and cosine functions?

   \[
   \text{DTFT} \quad \text{DTFS} \quad \text{CTFT} \quad \text{CTFS}
   \]

f. (1 pt) Let \( X_1 \) represent the transform of a signal \( x_1 \) and \( X_2 \) represent the transform of a signal \( x_2 \). How is the transform of \( y = x_1 \times x_2 \) related to \( X_1 \) and \( X_2 \)?

   \( Y \propto X_1 \ast X_2 \) (in the frequency domain, the transforms are convolved)

g. (1 pt) Why is Parseval’s theorem important for engineering applications?

   It can guide specifications and design of the frequency response of systems because it tells you where the signal power or energy is concentrated in the frequency domain.

h. (1 pt) Which property do all four transforms share when the signal is real-valued?

   The transforms of real signals have complex conjugate symmetry.

i. (1 pt) Which of the transforms can be used to synthesize the original signal exactly?

   \[
   \text{DTFT} \quad \text{DTFS} \quad \text{CTFT} \quad \text{CTFS}
   \]

j. (1 pt) Which of the transforms are nonperiodic?

   \[
   \text{DTFT} \quad \text{DTFS} \quad \text{CTFT} \quad \text{CTFS}
   \]

k. (1 pt) Which transforms can be applied to nonperiodic signals?

   \[
   \text{DTFT} \quad \text{DTFS} \quad \text{CTFT} \quad \text{CTFS}
   \]

l. (2 pts) Let \( X_1 \) represent the transform of a signal \( x_1 \) and \( X_2 \) represent the transform of a signal \( x_2 \). What is the transform of \( y = 5x_1 + 2x_2 \)?

   \( Y = 5X_1 + 2X_2 \)

m. (1 pt) Which aspects of the transform are not affected by a shift in time?

   \[
   \text{Magnitude} \quad \text{Phase} \quad \text{Real Part} \quad \text{Odd Part}
   \]
2. Matching Signals and Transforms (10 pts)

Ten discrete-time signals are shown in the left column of plots. The discrete-time Fourier transform was calculated using a windowed segment with 128 points, as shown in the right column of plots. Enter the transform number corresponding to each signal in the spaces provided at the bottom of the page.

Signal A: 8  Signal B: 1  Signal C: 10  Signal D: 9  Signal E: 6
Signal F: 2  Signal G: 4  Signal H: 3  Signal I: 7  Signal J: 5
3. **Fourier Signal Analysis and LTI Systems (13 pts)**
The bode plot of a continuous-time LTI system is shown below. The input signal to the system is \( x(t) \) and the output is \( y(t) \). All of the poles and zeros of the system are between 100 rad/s and 100 krad/s.

![Bode Plot](image)

a. (1 pt) What type of filter is this most similar to?

   - Lowpass
   - Highpass
   - Bandpass
   - Bandstop
   - Notch

b. (1 pt) What is (are) the cutoff frequency (frequencies)? Include units with your answer.

   \[ \omega_c = 12200 \text{ rad/s} \] (10-13 k rad/s acceptable)

c. (1 pt) Over what range of frequencies does this system amplify the input signal?

   Wherever the magnitude > 0 dB: \( \omega > 1180 \text{ rad/s} \) (1000 – 1500 rad/s acceptable)

d. (8 pts) Find the approximate output signal given the input signal,

   \[ x(t) = 43 + 29 \cos(1200t + 18^\circ) + 3.4 \sin(400t - 293^\circ) + 0.01 \cos(1,000,000t + 78^\circ) \]

   \[ y(t) = 0.041 + 32 \cos(1200t - 328^\circ) + 0.0048 \sin(400t - 194^\circ) + 100 \cos(1,000,000t + 76^\circ) \]

   (Coefficients only need to be accurate within reasonable the resolution of the plot.)

e. (1 pt) What is the fundamental period of the signal in the previous question?

   \[ T_o = \frac{2\pi}{400} = \frac{\pi}{200} \] (400 rad/s is the fundamental frequency of this periodic signal)

f. (1 pt) What is the approximate gain of the system at 100 Hz?

   \[ 100 \text{ Hz} = 2\pi \times 100 \text{ rad/s} \approx 628 \text{ rad/s} \]

   \[ |H(j628)| = 0.054 \] (-25.4 dB)
4. Applications (13 points)

a. (1 pt) Suppose a signal has a finite bandwidth such that its transform is approximately 0 for frequencies greater than $\omega_x$. Make up a magnitude spectrum for $x(t)$ and sketch it below for positive and negative frequencies. Label both axes.

![Magnitude Spectrum](image)

\[ |X(j\omega)| \]

\[ \omega \text{ (rad/s)} \]

b. (4 pts) Suppose this signal is multiplied by $\cos(\omega_o t)$ where $\omega_o > \omega_x$, $y(t) = \cos(\omega_o t) x(t)$. Sketch the magnitude spectrum of $y(t)$ below for positive and negative frequencies. Label both axes. Label $\omega_o$, $\omega_x$, and all other relevant frequencies.

![Magnitude Spectrum](image)

\[ |Y(j\omega)| \]

\[ \omega \text{ (rad/s)} \]

c. (2 pts) Suppose an ideal bandpass filter is applied to $y(t)$. What cutoff frequencies must the filter have to minimize the passband range without distorting $y(t)$?

$(\omega_o - \omega_x)$ and $(\omega_o + \omega_x)$

d. (1 pt) In most signal analysis applications, a signal is windowed and the CTFT is estimated. How can you determine if a periodic signal is present given the magnitude spectrum of the windowed signal?

Look for sharp, narrow peaks. If the harmonics are present, they will be at uniformly spaced frequencies.

e. (1 pt) How can you tell if the periodic signal in the previous question is sinusoidal or non-sinusoidal?

If it is sinusoidal, there will only be a single peak at the fundamental frequency. If the signal is non-sinusoidal, there will be harmonic peaks at integer multiples of the fundamental frequency as well.

f. (1 pt) Suppose the signal $x(t) = 3\exp(j(0.35t + 19^\circ))$ is applied to an LTI system with a transfer function represented by $H(j\omega)$. Write a symbolic expression for the output of the system. Note that this signal is complex-valued.

$y(t) = 3 \, H(j0.35) \, \exp(j(0.35t + 19^\circ))$

g. (3 pts) Suppose the signal $x(t) = 3\cos(0.35t + 19^\circ)$ is applied to an LTI system with a transfer function represented by $H(j\omega)$. Write a symbolic expression for the output of the system.

$y(t) = 3 \, |H(j0.35)| \, \cos(0.35t + 19^\circ + \ar{H(j0.35)})$