Final Exam Solutions
June 7, 2004

ECE 223: Signals & Systems II
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- Write your name above.
- Keep your exam flat during the entire exam period.
- If you have to leave the exam temporarily, close the exam and leave it face down while you are out of the room.
- Turn off any cell phones or pagers that might interrupt the exam.
- Do not open the exam until instructed to do so.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- You will have 100 minutes to complete the exam.
- If you have extra time, double check your answers.
- Remember to include units with each of your answers.

Problem 1:______ / 12
Problem 2:______ / 7
Problem 3:______ / 11
Problem 4:______ / 10
Problem 5:______ / 10

Total:______ / 50
1. **Foundation Concepts (12 pts)**

Circle the appropriate answers to the multiple choice questions below. The following abbreviations are used: DT = Discrete-Time, and CT = Continuous-Time.

a. (3 pts) Signals that are only scaled when they are applied to an LTI system are called eigenfunctions. Which of the following signals are eigenfunctions of LTI systems?

- $x(t) = \exp(-j\pi t)$
- $x(t) = \cos(500\pi t)$
- $x(t) = 5$
- $x[n] = \exp(-j\pi n)$
- $x[n] = \cos(500\pi n)$
- $x[n] = 5$

b. (1 pt) What is the magnitude of $x[n] = 9 \exp(-j(\pi n + 0.1\pi))$?

$|x[n]| = 9$

c. (1 pt) What is the fundamental period of $x[n] = \cos(0.2\pi n)$?

$N_o = 10$

d. (1 pt) What is the fundamental period of $x[n] = \cos(500\pi n)$?

$N_o = 1$

e. (1 pt) What is the fundamental period of $x(t) = \cos(0.2\pi t)$?

$T_o = 10$

f. (1 pt) Which of the following signals are periodic? Circle all that apply

- $x(t) = \cos(32t + 0.21\pi)$
- $x[n] = \sin(0.7\pi n + 0.3\pi)$
- $x[n] = \cos(32n + 0.21\pi)$


g. (1 pt) Consider a periodic DT signal with fundamental period $N$. How many distinct complex exponential harmonics could this signal have?

$N$

h. (1 pt) Consider a periodic CT signal with fundamental period $T$. How many distinct complex exponential harmonics could this signal have?

$\infty$

i. (1 pt) What is a simpler expression for $x[n] = \exp(-j5\pi n)$?

$x[n] = (-1)^n$

j. (1 pt) What is the fundamental period of $x[n] = \exp(-j5\pi n)$?

$N_o = 2$
2. **Sampling Fundamentals (7 pts)**

   a. (1 pt) If the signal \( x(t) = \cos(2000\pi t) \) is sampled at 5000 Hz such that \( x[n] = x(nT_s) \), what is the fundamental frequency of \( x[n] \) in radians/sample?

   \[ \Omega_o = \frac{2\pi}{5} \text{ radians/sample} \]

   b. (1 pt) Repeat question a. for a sampling frequency of 2000 Hz.

   \[ \Omega_o = \pi \text{ radians/sample} \]

   c. (1 pt) Repeat question a. for a sampling frequency of 8000 Hz.

   \[ \Omega_o = \frac{2\pi}{8} \text{ radians/sample} \]

   d. (1 pt) If the signal \( x(t) = \cos(2000\pi t) \) is sampled at 5000 Hz such that \( x[n] = x(nT_s) \), and \( x[n] \) is converted into a continuous-time signal through bandlimited interpolation, as described in class, what is the fundamental frequency of the reconstructed signal in units of Hz?

   \[ f_o = 1000 \text{ Hz} \]

   e. (1 pt) Repeat question d. for a sampling frequency of 2000 Hz.

   \[ f_o = 1000 \text{ Hz} \]

   f. (1 pt) Repeat question d. for a sampling frequency of 8000 Hz.

   \[ f_o = 1000 \text{ Hz} \]

   g. (1 pt) Why is ideal bandlimited interpolation impossible?

   *Ideal band-limited interpolation requires a filter that is non-causal and an infinite duration impulse response.*
3. **Sampling Applications (11 pts)**

An electrocardiogram is a voltage signal that normally has most of its power at frequencies less than 100 Hz. Occasionally the signal is corrupted by artifact than contains power at higher frequencies. Suppose the signal will be sampled at 250 Hz.

a. (1 pt) What type of filter must be used to prevent aliasing and to minimize the effects of artifact?

   Lowpass  Highpass  Bandpass  Bandstop  Notch

b. (1 pt) Can this filter be applied before sampling in continuous-time, after sampling in discrete-time, or either?

   Before  After  Either  Neither

c. (1 pt) What is the highest possible frequency that the stopband can begin without causing aliasing or distorting the signal?

   \[ f_{sb} = 125 \text{ Hz} \]

d. (1 pt) What is the lowest possible frequency that the passband can end without causing aliasing or distorting the signal?

   \[ f_{pb} = 100 \text{ Hz} \]

e. (1 pt) If the signal is also corrupted with 60 Hz electrical interference, what type of filter is most appropriate to eliminate this?

   Lowpass  Highpass  Bandpass  Bandstop  Notch

f. (1 pt) Can this filter in question e. be applied before sampling in continuous-time, after sampling in discrete-time, or either?

   Before  After  Either  Neither

Aliasing is known to occur in movies and on television with rotating objects, such as spoked wheels and helicopter blades, that repeat faster than the frame rate of the cameras. With newer digital cameras, each pixel can be represented as a signal that is sampled at the frame rate of the camera.

g. (1 pt) Suppose a light with sinusoidal illumination is recorded by a movie camera with a frame rate of 24 frames per second. How fast could the light oscillate before aliasing will occur?

   \[ f = 12 \text{ Hz} \]

h. (2 pts) Is it possible to eliminate aliasing by applying a separate antialiasing filter to each pixel prior to sampling? If so, explain why this would be difficult in practice. If not, explain why not.

   Yes, but it would be difficult in practice because a separate filter would be required for each pixel and movie cameras have very high resolutions (>2 million pixels)

i. (2 pts) Could this same technique be applied to older films? If so, explain why this would be difficult in practice. If not, explain why not.

   No, aliasing cannot be removed after sampling. In this case, sampling occurs as soon as the images are captured in each frame of the film.
4. Fourier Series and Transforms (10 pts)
Circle the appropriate answers to the multiple choice questions below. The following abbreviations are used: FT = Fourier Transform, FS = Fourier Series, DT = Discrete-Time, and CT = Continuous-Time, DFT = Discrete Fourier Transform, FFT = Fast Fourier Transform. Assume the signal is real-valued for all of these questions.

a. (1 pt) Which transform is actually calculated by hardware, such as digital oscilloscopes, to estimate the CTFT?
   - CTFT
   - CTFS
   - DTFS
   - DTFT
   - DFT

b. (1 pt) Which transform is exactly computed by the FFT?
   - CTFT
   - CTFS
   - DTFS
   - DTFT
   - DFT

c. (1 pt) If a finite segment of a DT signal contains 21 samples, how many zeros must be appended to the signal before the FFT can be applied?
   - 11 zeros

d. (2 pts) What problems with the FFT does zero-padding solve?
   - Signal duration must be an integer power of 2
   - The FFT only calculates the transform at \( N \) points, where \( N \) is the duration of the signal

e. (1 pt) What effect does windowing have on the spectrum of a signal?
   - It blurs the spectrum by convolving the window’s transform with the signal’s transform in the frequency domain

f. (1 pt) Why is windowing necessary in practical applications?
   - In practice, the transforms can only be calculated on signals with finite duration

g. (1 pt) Which of the transforms are periodic functions of frequency or a frequency index?
   - CTFT
   - CTFS
   - DTFT
   - DTFS

h. (1 pt) Which of the transforms have both a synthesis and analysis equation?
   - CTFT
   - CTFS
   - DTFT
   - DTFS

i. (1 pt) Which of the transforms require an infinite frequency range, in general?
   - CTFT
   - CTFS
   - DTFT
   - DTFS
5. Communications Concepts (10 pts)

a. (6 pts) Six signals are shown below. Match the signal with the type of signal. The following abbreviations are used: BBS = Baseband signal, CS = Carrier Signal, AAM = Asynchronous Amplitude Modulation, SAM = Synchronous Amplitude Modulation, FM = Frequency Modulation, PM = Phase Modulation.

Circle the appropriate answers to the multiple choice questions below.

b. (1 pt) Which type of modulation can be demodulated with an envelope detector and lowpass filter?
   - [ ] AAM  - SAM  - FM  - PM

c. (1 pt) Which type of modulation requires the carrier signal to demodulate?
   - [ ] AAM  - [ ] SAM

d. (1 pt) If there is a limit to the transmission power of the modulated signal, which type of modulation is able to dedicate more power to transmitting the baseband signal?
   - [ ] AAM  - [ ] SAM

e. (1 pt) If a channel has a bandwidth of 100 kHz, how many baseband signals, each with a bandwidth of 15 kHz, could be multiplexed into a single composite signal and transmitted through the channel?
   \[ M = 6 \]