Exam 1B Solutions
April 23, 2003

ECE 223: Signals and Systems II
Dr. McNames

- Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
- Do not open the exam until instructed to do so.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- You will have 100 minutes to complete the exam.
- If you have extra time, double check your answers.
- Remember to include units with each of your answers.
- You are not allowed to use a calculator during this exam.

Problem 1:______ / 12
Problem 2:______ / 12
Problem 3:______ / 16
Problem 4:______ / 10

Total:______ / 50

First Letter in Last Name:________________

6-Digit Identification Number:______________

Student Identification Number:______________
1. Fundamental Concepts (12 pts)
Circle the appropriate answers to the multiple choice questions below. Note that some questions may have more than one correct answer that must be circled.

a. (2 pts) Which of the following signals could be expressed as a Fourier series?

- Sawtooth
- Speech
- Unit step function
- 60 Hz Voltage

b. (1 pt) What is the magnitude of \( x(t) = 23 \exp(-j200\pi t + 2\pi/3) \)?

\[ |x(t)| = 23 \]

c. (1 pt) What is the fundamental period of \( x(t) = 53 \exp(-j200\pi t + 2\pi/3) \)?

\[ T_o = 1/100 = 10 \text{ ms} \]

d. (1 pt) Complex exponentials of the form \( z^n \) are eigenfunctions of LTI systems. For Fourier series analysis, what values of \( z \) are used? (Circle only one)

- Real
- Imaginary
- Positive
- Negative
- Even
- Unit Magnitude

\[ z^n \]

e. (1 pt) Complex exponentials of the form \( e^{st} \) are eigenfunctions of LTI systems. For Fourier series analysis, what values of \( s \) are used? (Circle only one)

- Real
- Imaginary
- Positive
- Negative
- Even
- Unit Magnitude

\[ e^{st} \]

f. (1 pt) How many unique discrete-time complex exponentials exist that are periodic with a period of \( N = 18 \)? Note that the fundamental period of each exponential may be less than 18.

1, 9, 12, 18, 36, \( \infty \)

g. (1 pt) How many unique continuous-time complex exponentials exist that are periodic with a period of \( T = 18 \)? Note that the fundamental period of each exponential may be less than 18.

1, 9, 12, 18, 36, \( \infty \)

h. (1 pt) What is the fundamental period of \( x[n] = \cos(\pi n/19 + \pi/5) \)? If the signal is not periodic, write N.A.

\[ N_o = 38 \]

i. (1 pt) What is the fundamental period of \( x[n] = \cos(5n + 2\pi/5) \)? If the signal is not periodic, write N.A.

\[ N_o = \text{N.A.} \]

j. (1 pt) What is the fundamental period of \( x(t) = \cos(5t + 2\pi/5) \)? If the signal is not periodic, write N.A.

\[ T_o = 2\pi/5 \]

k. (1 pt) What type of property do the Fourier series exponential coefficients have due to the assumption that the signal is real?

- Complex conjugate symmetry: \( a_k = a_k^* \)
2. Fourier Series (12 points)

Use the periodic signal shown below to answer the following questions.

![Signal Diagram]

a. (1 pt) What is the average (DC) value of the signal \( x(t) \)?  
   \[ a_0 = 0 \]

b. (1 pt) What type of symmetry does \( x(t) \) have? (circle one)
   
   [ ] Even  [ ] Odd  [ ] Both  [ ] Neither  [ ] Complex-conjugate

(c) (1 pt) What is the fundamental period of \( x(t) \)?  \( T_0 = 5 \) s

d. (1 pt) What is the fundamental frequency of \( x(t) \)?  \( f_0 = 1/5 = 0.2 \) Hz

e. (1 pt) What is the fundamental frequency of \( x(t) \)?  \( \omega_0 = 2\pi/5 = 0.4\pi \) rads/s

f. (1 pt) Does the Fourier series approximation equal \( x(t) \) at all values of \( t \)?  Yes  [ ] No

g. (1 pt) Solve for the exponential Fourier series coefficient \( a_0 \). Simplify your answer as much as possible.
   \[ a_0 = 0 \]

h. (5 pts) Solve for the exponential Fourier series coefficients \( a_k \). Simplify your answer as much as possible.

\[ a_k = B_k = \frac{2}{T} \int_0^{T/2} x(t) \cos(k \omega t) \, dt = \frac{2}{5} \int_0^{2.5} x(t) \cos(k \omega t) \, dt \]

\[ a_k = \frac{2}{5} \int_0^1 \cos(k \omega t) \, dt + \frac{2}{5} \int_2^{2.5} -2 \cos(k \omega t) \, dt \]

\[ a_k = \frac{2}{5} \int_0^1 \sin(k \omega t) \left[ - \frac{4}{5} \frac{1}{k \omega} \sin(k \omega t) \right]^{2.5}_0 \]

\[ a_k = \frac{2}{5} \frac{1}{k \cdot \frac{2\pi}{5}} \left[ \sin(k \frac{2\pi}{5}) - \sin(0) \right] - \frac{4}{5} \frac{1}{k \cdot \frac{2\pi}{5}} \left[ \sin(k \frac{2\pi}{5}) - \sin(k \frac{2\pi}{5}) \right] \]

\[ a_k = \frac{1}{k \pi} \sin(k0.4\pi) + \frac{1}{k \pi} \sin(k0.8\pi) \]
3. Discrete-Time Fourier Series (16 pts)

Use the signal below to answer the following questions.

![Signal Diagram]

a. (1 pt) What type of symmetry does $x[n]$ have? (circle one)
   - Even
   - Odd
   - Both
   - Neither
   - Complex-conjugate

b. (1 pt) What is the average (DC) value of the signal $x[n]$? 0

c. (1 pt) What is the fundamental period of $x[n]$? $N_0 = 5$ samples

d. (1 pt) What is the fundamental frequency of $x[n]$? $f_0 = 1/5 = 0.2$ cycles/sample

e. (1 pt) What is the fundamental frequency of $x[n]$? $\omega_0 = 2\pi/5 = 0.4\pi$ rads/sample

f. (3 pts) Which of the following are equal to the exponential Fourier series coefficient $a_1$?
   - $a_0$
   - $a_{-4}$
   - $a_{-16}$
   - $a_{-1}$
   - $a_{11}$
   - $a_{25}$

  

g. (2 pts) The first two fast Fourier transform (FFT) coefficients of $x[n]$ are given as follows. What are the discrete-time Fourier series coefficients? Hints: the FFT analysis equation is $X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$.

<table>
<thead>
<tr>
<th>$X(0)$</th>
<th>$X(1)$</th>
<th>$X(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0.26 + j0.36$</td>
<td>$4.7 + j1.5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0.052 + j0.073$</td>
<td>$0.95 + j0.31$</td>
</tr>
</tbody>
</table>

h. (6 pts) Given your answers above, use the properties of the discrete-time Fourier series to solve for the following coefficients. If you were unable to answer part g., use $a_0 = 5$, $a_1 = 4 + j3$, and $a_2 = 2 + j1$.

- $a_{-1} = 0.052 - j0.073$
- $a_{-2} = 0.95 - j0.31$
- $a_{-3} = 0.95 + j0.31$
- $a_{-37} = 0.95 - j0.31$
4. Fourier Series and LTI Systems (10 pts)

The bode plot of a continuous-time LTI system is shown below. The input signal to the system is $x(t)$ and the output is $y(t)$.

\begin{align*}
a. \quad \text{(2 pts) Find the output signal given } x(t) &= 4.3 \sin(200t - 102^\circ). \\
y(t) &= 447 \sin(200t - 2^\circ) \\
b. \quad \text{(2 pts) Find the output signal given } x(t) &= 43 \sin(7000t + 23^\circ). \\
y(t) &= 44 \sin(7000t + 196^\circ) \\
\end{align*}

Use the following input signal to answer the remaining questions.

\begin{align*}
\text{x(t)} &= 94 + 85 \sin(750t + 194^\circ) + 2000 \sin(30,000t - 59^\circ) \\
\end{align*}

c. \quad \text{(1 pt) What is the fundamental period of the signal?} \\
T_0 = 2\pi/750 = \pi/325 \text{ s}

d. \quad \text{(5 pts) Find the output signal.} \\
y(t) &= 956 + 84 \sin(750t + 49^\circ) + 1.6 \sin(30,000t + 32^\circ)