Exam 2 Solutions
May 15, 2002

ECE 223: Signals and Systems II
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• Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
• You are not allowed to use a calculator on this exam.
• Do not begin the exam until instructed to do so.
• You have 100 minutes to complete the exam.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages.

Problem 1:_____ / 10
Problem 2:_____ / 15
Problem 3:_____ / 10
Problem 4:_____ / 15

Total:_____ / 50

First Letter in Last Name:________________

6-Digit Identification Number:____________

Student Identification Number:____________
1. Matching Signals and Transforms (10 pts)
Ten discrete-time signals are shown in the left column of plots. Their transforms, shown in the right column of plots, were estimated by windowing 128 samples and applying the FFT with zero-padding. Enter the transform number corresponding to each signal in the spaces provided at the bottom of the page.

Signal A: 8  Signal B: 7  Signal C: 1  Signal D: 6  Signal E: 3
Signal F: 2  Signal G: 4  Signal H: 9  Signal I: 10  Signal J: 5
2. Fundamentals of Transforms & Sampling (15 pts)
Part of the real and imaginary components of the transform of a real-valued discrete-time signal $x[n]$ are shown below. Use these plots to answer the following questions.

- (6 pts) Use symmetry of the transform to draw the missing segments of these plots.
- (1 pt) Does the discrete time signal $x[n]$ have a DC component? (Circle one)
  - Yes
  - No
  - Insufficient Information
- (1 pt) What type of symmetry does the signal have? (Circle one)
  - Even
  - Odd
  - None
  - Insufficient Information
- (2 pts) What percentage of the signal energy is between the frequencies of 0 and $\frac{\pi}{2}$? 100 %
- (1 pt) Is the signal periodic? (Circle one)
  - Yes
  - No
  - Insufficient Information
- (1 pt) Does the transform of $x[n]$ converge? (Circle one)
  - Yes
  - No
  - Insufficient Information
- (3 pts) A continuous time signal $x(t)$ is sampled with the impulse train discussed in class $\phi(t)$ and a sampling rate of 20 Hz. The sampled signal is then applied to a filter with the transfer function shown below. Sketch the continuous-time Fourier transform (CTFT) of the sampled signal $x_\phi(t)$ and the CTFT of the filter output $y(t)$. Be certain to label the vertical tick marks of both plots.
3. Continuous-Time Fourier Transforms (10 points)
Use the signal and Fourier transforms plotted below to answer the following questions. Note that the imaginary component of \( Y(j\omega) \) is zero. Hint: there is more than one way to approach these problems. The following anti-derivative may help you: \( \int te^{at} dt = (\frac{1}{a})^2 e^{at} (at - 1) + C \).

![Graph of x(t) and Y(j\omega)]

a. (1 pt) Does the Fourier transform of \( x(t) \) exist technically? (Circle one)

- [ ] Yes
- [x] No
- [ ] Insufficient Information

b. (1 pt) Does the Fourier transform of \( Y(j\omega) \) exist technically? (Circle one)

- [x] Yes
- [ ] No
- [ ] Insufficient Information

c. (3 pts) Solve for the Fourier transform of \( x(t) \). Simplify your answer as much as possible.

\[
x(t) = -u(t+3) + u(t+1) + u(t-1) - u(t-3)
\]

\[
X(j\omega) = -\frac{e^{j3\omega} - e^{-j3\omega} + e^{j\omega} + e^{-j\omega}}{j\omega} = -\frac{2\cos(3\omega) + 2\cos(\omega)}{j\omega}
\]

\[
X(j\omega) = 2j\frac{\cos(3\omega) - \cos(\omega)}{\omega}
\]

d. (1 pt) How much energy does the signal \( x(t) \) have?

\[
W_x = 4
\]

e. (3 pts) Solve for the signal \( y(t) \). Simplify your answer as much as possible.

\[
Y(j\omega) = \delta(\omega - 3\pi) + \delta(\omega + 3\pi) + \frac{1}{2} p_{2\pi}(\omega) + \frac{1}{2} p_{\pi}(\omega)
\]

\[
y(t) = \frac{1}{\pi} \cos(3\pi t) + \frac{\sin(\pi t) + \sin(2\pi t)}{2\pi t}
\]

f. (1 pt) How much energy does the signal \( y(t) \) have?

\[
W_y = \infty : \text{The sinusoidal component has infinite energy}
\]
4. Discrete-Time Fourier Transforms (15 pts)

Simplify your answers to the following questions as much as possible.

a. (2 pts) Compute the Fourier transform of \( x[n] = u[n-5] - u[n-27] \).
\[
X(e^{j\omega}) = \frac{e^{-j5\omega} - e^{-j27\omega}}{1 - e^{-j\omega}}
\]

b. (2 pts) Compute the Fourier transform of \( x[n] = \) for \( 5 \leq n \leq 10 \) and \( x[n] = 0 \) otherwise.
\[
x[n] = p[n] - p_3[n]; \quad p_3[n] \Leftrightarrow \sin((N + 0.5)\omega) \\
X(e^{j\omega}) = \frac{\sin(10.5\omega) - \sin(4.5\omega)}{\sin(0.5\omega)}
\]

c. (2 pts) Compute the Fourier transform of \( x[n] = \text{sinc}(0.2n) \).
\[
X(e^{j\omega}) = \begin{cases} 
5 & |\omega| < 0.2\pi + 2\pi\ell \\
0 & \text{otherwise}
\end{cases}
\]

d. (2 pts) Compute the inverse Fourier transform of \( X(e^{j\omega}) = -2 + 4e^{-j\omega} \).
\[
x[n] = -2\delta[n] + 4\delta[n-1]
\]

e. (2 pts) Compute the inverse Fourier transform of \( X(e^{j\omega}) = 3\sum_{k=-\infty}^{\infty} \delta(\omega - \frac{\pi}{4}k) \).
\[
\sum_{k=-\infty}^{\infty} \delta[n-kN] \Leftrightarrow \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{N}) \\
\sum_{k=-\infty}^{\infty} \delta[n-k8] \Leftrightarrow 3\sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{8}) \\
x[n] = \frac{12}{\pi} \sum_{k=-\infty}^{\infty} \delta[n-k8] = \frac{3}{2\pi} \sum_{k=8}^{\infty} e^{j\frac{2\pi k}{8}}
\]

A causal and stable LTI system has an input \( x[n] \) and output \( y[n] \) that are related through the following difference equation: \( y[n] - 0.2y[n-1] = 0.8x[n] + 0.3x[n-1] \).

f. (2 pts) Determine the frequency response \( H(e^{j\omega}) \) of the system.
\[
Y(e^{j\omega}) - 0.2e^{-j\omega}Y(e^{j\omega}) = 0.8X(e^{j\omega}) + 0.3e^{-j\omega}X(e^{j\omega}) \\
H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{0.8 + 0.3e^{-j\omega}}{1 - 0.2e^{-j\omega}}
\]

g. (3 pts) Determine the impulse response of the system.
\[
H(e^{j\omega}) = \frac{0.8}{1-0.2e^{-j\omega}} + \frac{0.3e^{-j\omega}}{1-0.2e^{-j\omega}} \Leftrightarrow 0.8 \cdot (0.2)^nu[n] + 0.3 \cdot (0.2)^{-1}u[n-1] \\
h[n] = 0.8 \cdot (0.2)^nu[n] + 0.3 \cdot (0.2)^{-1}u[n-1]
\]