ECE 223: Signals and Systems II
Dr. McNames

- Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
- You are not allowed to use a calculator on this exam.
- Do not begin the exam until instructed to do so.
- You have 100 minutes to complete the exam.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages.

Problem 1: _____ / 11
Problem 2: _____ / 12
Problem 3: _____ / 11
Problem 4: _____ / 16

Total: _____ / 50

First Letter in Last Name: _______________
6-Digit Identification Number: _______________
Student Identification Number: _______________
1. Fundamental Concepts (11 pts)

a. (1 pt) What do the letters LTI stand for in this course?

\[ \text{Linear & Time Invariant} \]

b. (1 pt) Eigenfunctions and eigenvalues are properties of signals, not systems.

True [ ] False [X]

c. (1 pt) Sums of complex exponentials can be used to represent any periodic voltage or current signal that could be produced by a real function generator.

True [X] False [ ]

d. (1 pt) What is the fundamental period of \( x(t) = 3 - 3 \cos(\pi/6\,t) + 17 \cos(\pi/9\,t) \)? Write \( \infty \) if the signal is not periodic.

\[ o_T = 24 \, \text{s} \]

e. (1 pt) What is the fundamental period of \( x[n] = 3 + 7u(n) + \cos(\pi/4\,n) + 17 \cos(\pi/9\,n) \)? Write \( \infty \) if the signal is not periodic.

\[ N_o = \infty \]

f. (1 pt) If the signal \( x(t) = e^{-at} \) is applied to an LTI system, the output will be proportional to the input signal regardless of the value of \( s \).

True [X] False [ ]

g. (1 pt) The signals \( x_1[n] = e^{-3\omega n} \) and \( x_2[n] = e^{5\omega n} \) are equal for all values of \( n \) if \( \omega = \frac{2\pi}{5} \).

True [X] False [ ]

h. (1 pt) The integral of an even periodic signal over one fundamental period, \( \int_{o} x_e(t)dt \), is equal to zero.

True [ ] False [X] : In general

i. (1 pt) If a signal \( x(t) \) has odd symmetry, we know that it must be zero when \( t = 0 \).

True [X] False [ ]

j. (1 pt) Any finite-valued periodic discrete-time signal can be exactly represented as a sum of \( n \) harmonically related complex exponentials.

True [X] False [ ]

k. (1 pt) Discontinuities in continuous-time signals cause Gibb’s phenomenon in the Fourier series representation of the signals.

True [X] False [ ]
2. Fourier Series (12 points)

Use the periodic signal shown below to answer the following questions. Hints:
\[ \int \sin(\omega t)dt = \frac{1}{\omega} \sin(\omega t) - \frac{t}{\omega} \cos(\omega t) + C, \quad \int \cos(\omega t)dt = \frac{1}{\omega^2} \cos(\omega t) + \frac{t}{\omega} \sin(\omega t) + C. \]

\[ x(t) \]
\[ t \text{ (seconds)} \]
\[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]
\[ -1 \quad 0 \quad 0.5 \quad 1 \]

a. (1 pt) Does \( x(t) \) have even symmetry (circle one)? Yes \[\square\] No

b. (1 pt) Does \( x(t) \) have odd symmetry (circle one)? Yes \[\square\] No

c. (1 pt) What is the fundamental period of \( x(t) \)? 4 s

d. (1 pt) What is the fundamental frequency of \( x(t) \)? 0.25 Hz

e. (1 pt) What is the fundamental frequency of \( x(t) \)? 0.5 \( \pi \) rad/s

f. (1 pt) Does the Fourier series approximation equal \( x(t) \) at all values of \( t \)? Yes \[\square\] No

g. (1 pt) The Fourier series can be written in trigonometric form,
\[ x(t) = a_0 + 2 \sum_{k=1}^{\infty} B_k \cos(k \omega_0 t) - C_k \sin(k \omega_0 t). \]
Solve for the coefficient \( a_0 \). Simplify your answer as much as possible.

\[ a_0 = 0 \] since the average over one period is zero

h. (5 pts) Solve for the coefficients \( B_k \) and \( C_k \). If possible, use symmetry to make this task easier. Simplify your answer as much as possible.

\[ B_k = 0 \] since the signal has odd symmetry

\[ C_k = -\frac{2}{T} \int_0^T x(t) \sin(k \omega_0 t)dt \]

\[ C_k = -\frac{1}{2} \left[ \int_0^t -t \sin(\omega_0 t)dt - \frac{1}{2} \int_1^2 \sin(\omega_0 t)dt \right] \]

\[ C_k = \frac{1}{2} \left[ \sin(\omega_0) - \frac{1}{k \omega} \cos(\omega_0) + \frac{1}{2} \frac{1}{k \omega} \cos(\omega_0) \right]^2 \]

\[ C_k = \frac{1}{2} \left[ \sin(\omega_0) - \frac{1}{k \omega} \cos(\omega_0) + \frac{1}{2} \frac{1}{k \omega} \cos(\omega_0) \right] + \frac{1}{2} \frac{1}{k \omega} \cos(2 \omega) \]

\[ C_k = \frac{1}{2} \left( \frac{1}{k \omega} \right) \sin(k \omega_0) - \frac{1}{2} \frac{1}{k \omega} \cos(k \omega) \]

\[ C_k = \frac{1}{2} \left( \frac{1}{k \omega} \right) \sin(k \omega_0) - \frac{1}{2} \frac{1}{k \omega} \cos(k \omega) + \frac{1}{2} \frac{1}{k \omega} \cos(2k \omega) \]

\[ C_k = \frac{1}{2} \left( \frac{1}{k \omega} \right) \sin(k \omega_0) - \frac{1}{2} \frac{1}{k \omega} \cos(k \omega) + \frac{1}{k \omega} (-1)^k \]
3. Fourier Series and LTI Systems (11 pts)
The bode plot of a continuous-time LTI system is shown below. The input signal to the system is \( x(t) \) and the output is \( y(t) \).

\[
\text{Mag (dB)}
\]
\[
\text{Phase (deg)}
\]

\[
\begin{array}{c}
10^{-1} \\
10^{-2} \\
10^{-3} \\
10^{-4} \\
10^{-5}
\end{array}
\]
\[
\begin{array}{c}
10^1 \\
10^2 \\
10^3 \\
10^4 \\
10^5
\end{array}
\]

\[
\begin{array}{c}
10 \\
10^2 \\
10^3 \\
10^4 \\
10^5
\end{array}
\]
\[
\begin{array}{c}
-200 \\
-150 \\
-100 \\
-50 \\
0
\end{array}
\]
\[
\begin{array}{c}
200 \\
150 \\
100 \\
50 \\
0
\end{array}
\]

\[
\begin{array}{c}
20 \\
10 \\
0 \\
-10 \\
-20
\end{array}
\]
\[
\begin{array}{c}
20 \\
10 \\
0 \\
-10 \\
-20
\end{array}
\]

\[
\begin{array}{c}
30 \\
40 \\
50 \\
60 \\
70
\end{array}
\]
\[
\begin{array}{c}
30 \\
40 \\
50 \\
60 \\
70
\end{array}
\]

\[
\begin{array}{c}
80 \\
90 \\
100 \\
110 \\
120
\end{array}
\]
\[
\begin{array}{c}
80 \\
90 \\
100 \\
110 \\
120
\end{array}
\]

\[
\begin{array}{c}
0 \\
-10 \\
-20 \\
-30 \\
-40
\end{array}
\]
\[
\begin{array}{c}
0 \\
-10 \\
-20 \\
-30 \\
-40
\end{array}
\]

\[
\begin{array}{c}
0 \\
-10 \\
-20 \\
-30 \\
-40
\end{array}
\]
\[
\begin{array}{c}
0 \\
-10 \\
-20 \\
-30 \\
-40
\end{array}
\]

a. (2 pts) Find the output signal given \( x(t) = 27 \sin(100t + 32^\circ) \).

\[
y(t) = 2.76 \sin(100t + 77.6^\circ)
\]

b. (2 pts) Find the output signal given \( x(t) = 73 \cos(40,000t - 23^\circ) \).

\[
y(t) = 0.700 \cos(40,000t - 139^\circ)
\]

Use the following input signal to answer the remaining questions.

\[
x(t) = 12 \cos(1000t - 18^\circ) + 38 \sin(7000t + 83^\circ) + 94 \cos(40,000t + 120^\circ)
\]

c. (1 pt) What is the fundamental period of the signal?

\[
T_o = \frac{\pi}{500} \text{ s}
\]

d. (6 pts) Find the output signal.

\[
y(t) = 116 \cos(1000t - 13.7^\circ) + 36 \sin(7000t - 27^\circ) + 0.90 \cos(40,000t + 3.6^\circ)
\]

Note: approximate values are acceptable for this problem.
4. Discrete-Time Fourier Series (16 pts)
The input signal to a discrete-time LTI system is given below.
\[ x[n] = -2 + 4\cos\left(\frac{\pi}{3} n\right) + 6\sin\left(\frac{4\pi}{9} n\right) - 2\cos\left(\frac{14\pi}{6} n\right) - 4\sin\left(\frac{\pi}{3} n\right) \]

a. (1 pt) What is the fundamental period of \( x[n] \)? \( N_o = 18 \)

b. (1 pt) Is the following expression true: \( x[n] = x[n+9] \)? \[ \text{Yes} \quad \text{No} \]

c. (1 pt) Is the following expression true: \( x[n] = x[n-36] \)? \[ \text{Yes} \quad \text{No} \]

d. (1 pt) Is the following expression true: \( a_i = a_{i9} \)? \[ \text{Yes} \quad \text{No} \]

e. (1 pt) Is the following expression true: \( a_5 = a_{-5}^* \)? \[ \text{Yes} \quad \text{No} \]

f. (5 pts) Write \( x[n] \) as a linear combination of the \( N_o \) harmonics closest to the fundamental.

For example, \( 8\cos(2\pi/5) \) would be written as \( 4e^{j\frac{2\pi}{5} n} + 4e^{-j\frac{2\pi}{5} n} \).
\[ x[n] = -2 + 4\cos\left(\frac{2\pi}{6} n\right) + 6\sin\left(\frac{2\pi}{9} n\right) - 2\cos\left(\frac{12\pi}{6} n + \frac{2\pi}{6} n\right) - 4\sin\left(\frac{2\pi}{6} n\right) \]
\[ x[n] = -2 + 4\cos\left(\frac{3\pi}{18} n\right) + 6\sin\left(\frac{4\pi}{18} n\right) - 2\cos\left(\frac{2\pi}{18} n + \frac{2\pi}{18} n\right) - 4\sin\left(\frac{2\pi}{18} n\right) \]
\[ x[n] = -2 + 2\cos\left(\frac{3\pi}{18} n\right) + 6\sin\left(\frac{4\pi}{18} n\right) - 4\sin\left(\frac{2\pi}{18} n\right) \]
\[ x[n] = -2 + \frac{2}{j2}\left(e^{j\frac{3\pi}{18} n} + e^{-j\frac{3\pi}{18} n}\right) + \frac{6}{j2}\left(e^{j\frac{4\pi}{18} n} - e^{-j\frac{4\pi}{18} n}\right) - \frac{4}{j2}\left(e^{j\frac{2\pi}{18} n} - e^{-j\frac{2\pi}{18} n}\right) \]
\[ x[n] = -2 + \left(e^{j\frac{3\pi}{18} n} + e^{-j\frac{3\pi}{18} n}\right) - 3j\left(e^{j\frac{4\pi}{18} n} - e^{-j\frac{4\pi}{18} n}\right) + 2j\left(e^{j\frac{2\pi}{18} n} - e^{-j\frac{2\pi}{18} n}\right) \]

g. (6 pts) Use the axes below to draw stem plots for all the non-zero discrete-time Fourier series coefficients \( a_k \). Use the top axis to plot the real part and the bottom axis for the imaginary part of each coefficient.