Exam 1
April 22, 2002

ECE 223: Signals and Systems II
Dr. McNames

• Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
• You are not allowed to use a calculator on this exam.
• Do not begin the exam until instructed to do so.
• You have 100 minutes to complete the exam.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages.

Problem 1:_____ / 11
Problem 2:_____ / 12
Problem 3:_____ / 11
Problem 4:_____ / 16

Total:_____ / 50

First Letter in Last Name:____________________
6-Digit Identification Number:________________
Student Identification Number:________________
1. Fundamental Concepts (11 pts)

a. (1 pt) What do the letters LTI stand for in this course?

__________________________________

b. (1 pt) Eigenfunctions and eigenvalues are properties of signals, not systems.

True    False

c. (1 pt) Sums of complex exponentials can be used to represent any periodic voltage or current signal that could be produced by a real function generator.

True    False

d. (1 pt) What is the fundamental period of

\[ x(t) = 3 - 3\cos(\pi/3t) + 17\cos(\pi/6t) \] ? Write \( \infty \) if the signal is not periodic.

\[ T_o = \]

e. (1 pt) What is the fundamental period of

\[ x[n] = 3 + 7u(n) + \cos(\pi/4n) + 17\cos(\pi/2n) \] ? Write \( \infty \) if the signal is not periodic.

\[ N_o = \]

f. (1 pt) If the signal \( x(t) = e^{-st} \) is applied to an LTI system, the output will be proportional to the input signal regardless of the value of \( s \).

True    False

g. (1 pt) The signals \( x_1[n] = e^{-3in} \) and \( x_2[n] = e^{5in} \) are equal for all values of \( n \) if \( \omega = \frac{2\pi}{8} \).

True    False

h. (1 pt) The integral of an even periodic signal over one fundamental period, \( \int_{-T_o}^{T_o} x_e(t) \, dt \), is equal to zero.

True    False

i. (1 pt) If a signal \( x(t) \) has odd symmetry, we know that it must be zero when \( t = 0 \).

True    False

j. (1 pt) Any finite-valued periodic discrete-time signal can be exactly represented as a sum of \( n \) harmonically related complex exponentials.

True    False

k. (1 pt) Discontinuities in continuous-time signals cause Gibb’s phenomenon in the Fourier series representation of the signals.

True    False
2. Fourier Series (12 points)
Use the periodic signal shown below to answer the following questions. Hints:
\[ \int t \sin(\omega t) dt = \frac{1}{\omega} \sin(\omega t) - \frac{t}{\omega} \cos(\omega t) + C, \quad \int t \cos(\omega t) dt = \frac{1}{\omega^2} \cos(\omega t) + \frac{t}{\omega} \sin(\omega t) + C. \]

a. (1 pt) Does \( x(t) \) have even symmetry (circle one)?  
   Yes  
   No
b. (1 pt) Does \( x(t) \) have odd symmetry (circle one)?  
   Yes  
   No
c. (1 pt) What is the fundamental period of \( x(t) \)?  
   _________ s

d. (1 pt) What is the fundamental frequency of \( x(t) \)?  
   _________ Hz
e. (1 pt) What is the fundamental frequency of \( x(t) \)?  
   _________ rad/s
f. (1 pt) Does the Fourier series approximation equal \( x(t) \) at all values of \( t \)?  
   Yes  
   No
g. (1 pt) The Fourier series can be written in trigonometric form,
   \[ x(t) = a_0 + 2 \sum_{k=1}^{\infty} B_k \cos(k \omega_o t) - C_k \sin(k \omega_o t). \]
   Solve for the coefficient \( a_0 \). Simplify your answer as much as possible.

\[ a_0 = \]
h. (5 pts) Solve for the coefficients \( B_k \) and \( C_k \). If possible, use symmetry to make this task easier. Simplify your answer as much as possible.

\[ B_k = \]
\[ C_k = \]
3. Fourier Series and LTI Systems (11 pts)
The bode plot of a continuous-time LTI system is shown below. The input signal to the system is $x(t)$ and the output is $y(t)$.

![Bode Plot](image)

a. (2 pts) Find the output signal given $x(t) = 27 \sin(100t + 32^\circ)$.

$y(t) =$

b. (2 pts) Find the output signal given $x(t) = 73 \cos(40,000t - 23^\circ)$.

$y(t) =$

Use the following input signal to answer the remaining questions.

$x(t) = 12 \cos(1000t - 18^\circ) + 38 \sin(7000t + 83^\circ) + 94 \cos(40,000t + 120^\circ)$

c. (1 pt) What is the fundamental period of the signal?

$T_o =$

d. (6 pts) Find the output signal.

$y(t) =$
4. Discrete-Time Fourier Series (16 pts)

The input signal to a discrete-time LTI system is given below.

\[ x[n] = -2 + 4 \cos \left( \frac{\pi}{3} n \right) + 6 \sin \left( \frac{4\pi}{9} n \right) - 2 \cos \left( \frac{14\pi}{6} n \right) - 4 \sin \left( \frac{\pi}{3} n \right) \]

a. (1 pt) What is the fundamental period of \( x[n] \)? \( N_o = \) ________

b. (1 pt) Is the following expression true: \( x[n] = x[n + 9] \)? Yes No

c. (1 pt) Is the following expression true: \( x[n] = x[n - 36] \)? Yes No

d. (1 pt) Is the following expression true: \( a_i = a_{i9} \)? Yes No

e. (1 pt) Is the following expression true: \( a_s = a_s^* \)? Yes No

f. (5 pts) Write \( x[n] \) as a linear combination of the \( N_o \) harmonics closest to the fundamental.

For example, \( 8 \cos(2\pi/5) \) would be written as \( 4e^{j\pi n/5} + 4e^{-j\pi n/5} \).

\[ x[n] = \]

g. (6 pts) Use the axes below to draw stem plots for all the non-zero discrete-time Fourier series coefficients \( a_k \). Use the top axis to plot the real part and the bottom axis for the imaginary part of each coefficient.