1. Interpretation of the transforms.

a. How can you tell from the magnitude of the transforms what the DC component of the signal is?

The “DC component” is just the average value of the signal and is represented by either the value of the Fourier series coefficient at \( k = 0 \) or the area of the impulse located at a frequency of 0 for the Fourier transforms.

b. Which transforms can be calculated on a computer exactly, ignoring finite precision effects?

Only the Fourier series because they only require that the signal be known for a finite interval. The Fourier transforms cannot be calculated exactly for real signals because they require knowledge of the signal over an infinite time span.

c. Why do we represent signals as linear combinations of complex sinusoids rather than as real sinusoids?

Primarily because the math is easier and more elegant when working with complex sinusoids. If we use the amplitude-phase forms of the Fourier transforms we must evaluate two integrals to solve for the transforms and rely on trigonometric identities. However, real signals can be represented as either a linear combination of complex sinusoids with the complex conjugate symmetry to ensure the synthesized signals are real-valued or as a linear combination of sinusoids with the appropriate amplitudes and phases. The two representations are mathematically equivalent.

2. Relationship to CTFS Coefficients. Suppose that we have a periodic signal \( x(t) \) with fundamental period \( T \). Define the truncated signal \( x_T(t) \) as follows.

\[
x_T(t) = \begin{cases} x(t) & |t| < \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}
\]

a. Determine how the Fourier transform of \( x_T(t) \) is related to the continuous-time Fourier series coefficients of \( x(t) \).
b. Can the CTFT be used to calculate the CTFS coefficients of a periodic signal? If so, how?

Yes. Simply calculate the CTFT of one period of the periodic signal $x(t)$, scale by $\frac{1}{T}$ and evaluate the CTFT at integer multiples of the fundamental frequency, $\omega = k\omega_0$.

c. Can the CTFS be used to calculate the CTFT of a signal with finite duration? If so, at what frequencies?

Yes, but only at frequencies that are integer multiples of $\frac{2\pi}{T}$, where $T$ is the duration of the signal.

$$X(j\omega)|_{\omega = k\frac{2\pi}{T}} = TX[k]$$

3. Fourier properties concepts.

a. Suppose a music signal is bandlimited from 50 Hz to 20 kHz. If you wish to put this through a channel that is bandlimited from 0 to 5 kHz, how much would you have to stretch the signal in time?

To decrease the bandwidth by a factor of 4 to 5 kHz you would have to stretch the signal in time, $x(t/4)$ by a factor of 4.

b. What is the practical relevance of Parseval’s theorem?

Parseval’s theorem tells us that we can think of the Fourier transforms as either power or energy densities versus frequency. Practically this is useful because we can calculate what fraction of the signal power is located within a given frequency range. This is useful for design because if we are designing or applying a filter to our signal Parseval’s theorem can be used to compute what fraction of the signal power or energy will pass through the filter.

c. Why do we primarily use the Laplace transform to analyze the effect of LTI systems on signals instead of the Fourier transform?

There are primarily two reasons. First, the Laplace transform converges for a wider class of signals than the Fourier transform. Second, the Laplace transform is easier to work with mathematically than the Fourier transform because most of the algebraic expressions for $s$ have real coefficients whereas the coefficients of $\omega$ when using the Fourier transform are complex-valued, in general.

a. What is an ideal window?
   \[ w(t) = 1. \]

b. Why can’t it be used in practical applications such as integration with oscilloscopes?
   \textit{Because practical windows must have a finite duration: } \[ w(t) = 0 \text{ for } |t| > T. \]

c. What effect does using a finite window have on the spectrum?
   \textit{It smoothes or blurs the spectrum much like a lowpass filter would smooth or blur a signal in the time domain. This makes it difficult to identify sharp features in the spectrum such as impulses and abrupt edges.}