Exam 2
May 15, 2006

ECE 223: Signals & Systems II
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- Keep your exam flat during the entire exam.
- If you have to leave the exam temporarily, close the exam and leave it face down while you are out of the room.
- Turn off cell phones.
- Do not open the exam until instructed to do so.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- You will have 100 minutes to complete the exam.
- If you have extra time, double check your answers.
- Include units with each of your answers.
- Show all of your work for full credit.

Problem 1: _____ / 13
Problem 2: _____ / 10
Problem 3: _____ / 14
Problem 4: _____ / 13

Total: _____ / 50
1. Transform Properties (13 pts)
The following abbreviations are used throughout this exam: FT = Fourier Transform, FS = Fourier Series, DT = Discrete-Time, and CT = Continuous-Time. Unless specified otherwise, you may assume the signal is real-valued for all of these questions.

a. (1 pt) Which transforms can be represented by a finite frequency range?
   - DTFT   DTFS   CTFT   CTFS

b. (1 pt) Which transforms are periodic functions of frequency or a frequency index?
   - DTFT   DTFS   CTFT   CTFS

c. (1 pt) Which transforms are linear?
   - DTFT   DTFS   CTFT   CTFS

d. (1 pt) Which transforms always converge?
   - DTFT   DTFS   CTFT   CTFS

e. (1 pt) Which transforms can only be applied to periodic signals?
   - DTFT   DTFS   CTFT   CTFS

f. (1 pt) Which transforms converge when applied to periodic signals?
   - DTFT   DTFS   CTFT   CTFS

g. (1 pt) If a transform is finite for all frequencies, which of the following have synthesis equations that are guaranteed to converge?
   - DTFT   DTFS   CTFT   CTFS

h. (1 pt) Which of the following components are unaffected if the signal is shifted in time?
   - Real   Imaginary   Amplitude   Phase

i. (1 pt) Which of the following components are linear functions of the signals?
   - Real   Imaginary   Amplitude   Phase

j. (1 pt) Which of the following components represent the odd part of the signal?
   - Real   Imaginary   Amplitude   Phase

k. (1 pt) Which of the following components of a transform are even functions of frequency or a frequency index?
   - Real   Imaginary   Amplitude   Phase

l. (1 pt) Which of the following components of a transform are non-negative?
   - Real   Imaginary   Amplitude   Phase

m. (1 pt) Which of the following components of a transform are odd functions of frequency or a frequency index?
   - Real   Imaginary   Amplitude   Phase
2. Matching Signals and Transforms (10 pts)
Ten discrete-time signals are shown in the left column of plots. The discrete-time Fourier transform (DTFT) was calculated using a windowed segment with 128 points, as shown in the right column of plots. Enter the transform number corresponding to each signal in the spaces provided at the bottom of the page.
3. Discrete-Time Fourier Transform (14 pts)
   a. (4 pts) Suppose a real-valued discrete-time signal has a transform described by the following equation. Write similar expressions for the magnitude and phase (in degrees) of this transform.

   \[
   X(e^{j\Omega}) = \begin{cases} 
   -2j & 0 < \Omega \leq \frac{\pi}{4} \\
   \frac{1+j}{\sqrt{2}} & \frac{\pi}{4} < \Omega \leq \frac{\pi}{2} \\
   1 & \frac{\pi}{2} < \Omega \leq \frac{3\pi}{4} \\
   0 & \frac{3\pi}{4} < \Omega \leq \pi 
   \end{cases}
   \]

   \[
   |X(e^{j\Omega})| = \begin{cases} 
   0 & 0 < \Omega \leq \frac{\pi}{4} \\
   \frac{\sqrt{2}}{2} & \frac{\pi}{4} < \Omega \leq \frac{\pi}{2} \\
   \frac{1}{\sqrt{2}} & \frac{\pi}{2} < \Omega \leq \frac{3\pi}{4} \\
   0 & \frac{3\pi}{4} < \Omega \leq \pi 
   \end{cases}
   \]

   \[
   \angle X(e^{j\Omega}) = \begin{cases} 
   0 & 0 < \Omega \leq \frac{\pi}{4} \\
   \frac{\pi}{4} & \frac{\pi}{4} < \Omega \leq \frac{\pi}{2} \\
   \frac{\pi}{2} & \frac{\pi}{2} < \Omega \leq \frac{3\pi}{4} \\
   \frac{3\pi}{4} & \frac{3\pi}{4} < \Omega \leq \pi 
   \end{cases}
   \]

   b. (6 pts) Plot the magnitude and phase of this transform over the entire frequency range of the axes provided below.

   ![Magnitude and Phase Plot](image)

   c. (2 pts) What percentage of the signal energy is between the frequencies of 0 and \( \pi/2 \)? Hint: use Parseval’s theorem.

   d. (2 pts) What is the magnitude and phase of the transform at \( 52.7\pi/2 \)?

   \[
   |X(e^{j52.7\pi/2})| = \quad \angle X(e^{j52.7\pi/2}) =
   \]
4. Continuous Time Fourier Transform (13 pts)

a. (1 pt) Suppose a causal LTI system has an impulse response \( h(t) \). What is the difference between the Laplace transform of the impulse response \( H(s) \) evaluated at \( s=j\omega \) and the Fourier transform of the impulse response?

b. (1 pt) Why must the continuous-time Fourier transform be written as an integral with an infinite range of frequencies, rather than a finite range like the discrete-time Fourier transform?

c. (1 pt) Does the synthesis equation for the continuous-time Fourier transform always reproduce the original signal exactly? Explain.

Consider the following signal for the remaining questions.

\[ x(t) = \begin{cases} 
\cos(14\pi t) & 2 \leq |t| \leq 3 \\
0 & \text{Otherwise} 
\end{cases} \]

d. (2 pts) Sketch a time domain plot of this signal.

e. (1 pt) What type of signal is this?

- Energy
- Power
- Neither
- Both

f. (1 pt) What type of symmetry does this signal have?

- Even
- Odd
- None
- Both

g. (3 pts) List three properties that the continuous-time Fourier transform of this signal has.

h. (3 pts) Solve for the continuous-time Fourier transform of this signal. Hint: use known transforms and the transform properties, rather than applying the definition directly. Simplify your answer as much as possible

\[ X(j\omega) = \]