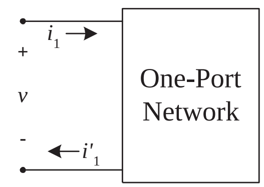
Two-Port Networks

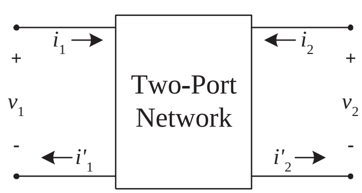
- Definitions
- Impedance Parameters
- Admittance Parameters
- Hybrid Parameters
- Transmission Parameters
- Cascaded Two-Port Networks
- Examples
- Applications

One-Port Networks



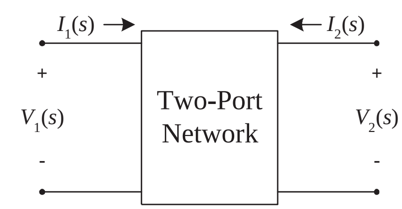
- A pair of terminals at which a signal (voltage or current) may enter or leave is called a **port**
- A network having only one such pair of terminals is called a **one-port network**
- No connections may be made to any other nodes internal to the network
- By KCL, we therefore have $i_1 = i_1'$
- We discussed in ECE 221 how one-port networks may be modeled by their Thévenin or Norton equivalents

Two-Port Networks: Definitions & Requirements



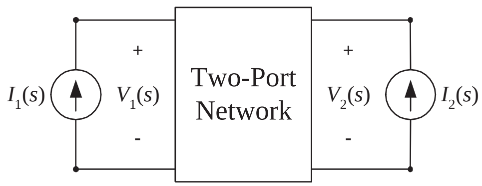
- Two-port networks are used to describe the relationship between a pair of terminals
- The analysis methods we will discuss require the following conditions be met
  1. Linearity
  2. No independent sources inside the network
  3. No stored energy inside the network (zero initial conditions)
  4. $i_1 = i_1'$ and $i_2 = i_2'$

Two-Port Networks: Defining Equations



- If the network contains dependent sources, one or more of the equivalent resistors may be negative
- Generally, the network is analyzed in the s domain
- Each two-port has exactly two governing equations that can be written in terms of any pair of network variables
- Like Thévenin and Norton equivalents of one-ports, once we know a set of governing equations we no longer need to know what is inside the box

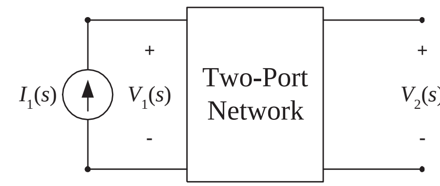
Impedance Parameters



$$
\begin{aligned}
V_1 &= z_{11}I_1 + z_{12}I_2 \\
V_2 &= z_{21}I_1 + z_{22}I_2
\end{aligned}
\qquad
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}
$$

- Suppose the currents and voltages can be measured
- Alternatively, if the circuit in the box is known, V_1 and V_2 can be calculated based on circuit analysis
- Relationship can be written in terms of the **impedance** parameters
- We can also calculate the impedance parameters after making two sets of measurements

Impedance Parameter Measurements

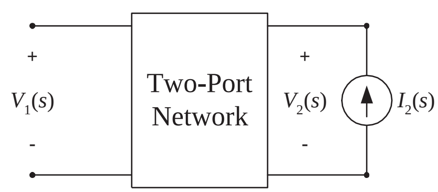


$$
\begin{aligned}
V_1 &= z_{11}I_1 + z_{12}I_2 \\
V_2 &= z_{21}I_1 + z_{22}I_2
\end{aligned}
$$

If the right port is an open circuit ($I_2 = 0$), then we can easily solve for two of the impedance parameters:

$$
z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}
$$

Impedance Parameter Measurements Continued

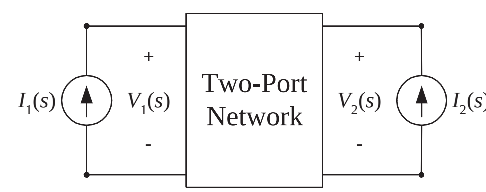


$$
\begin{aligned}
V_1 &= z_{11}I_1 + z_{12}I_2 \\
V_2 &= z_{21}I_1 + z_{22}I_2
\end{aligned}
$$

If the left port is an open circuit ($I_1 = 0$), then we can easily solve for the other two impedance parameters:

$$
z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}
$$

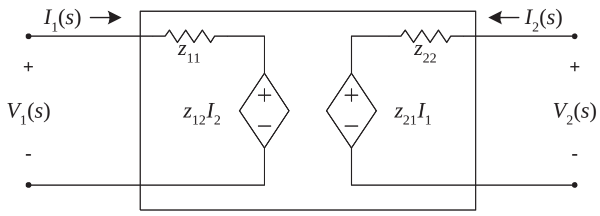
Impedance Parameter Measurements Summarized



$$
z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}
$$

$$
z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}
$$

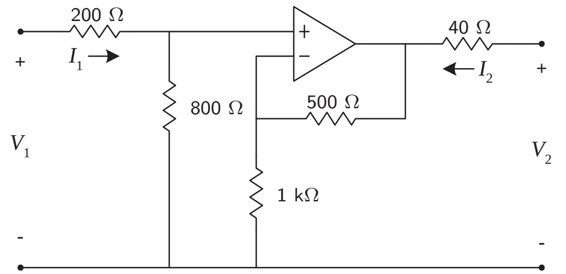
Impedance Parameter Equivalent



$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

- Once we know what the impedance parameters are, we can model the behavior of the two-port with an equivalent circuit.
- Notice the similarity to Thévenin and Norton equivalents

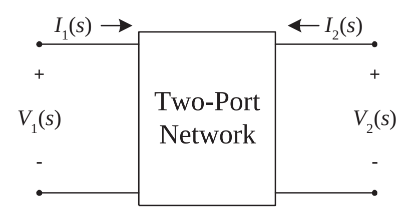
Example 1: Impedance Parameters



Find the z parameters of the circuit.

Example 1: Workspace

Example 2: Parameter Conversion



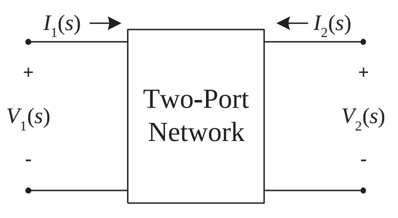
$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

In general, the two defining equations can be written in terms of any pair of variables. For example, rewrite the defining equations in terms of the voltages V_1 and V_2.

Example 2: Workspace

Example 2: Workspace Continued

Impedance & Admittance Parameters



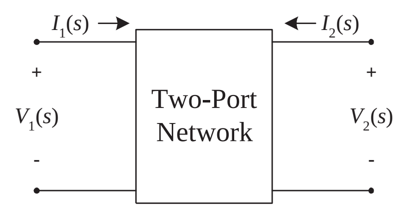
Impedance Parameters

$$V_1 = z_{11}I_1 + z_{12}I_2 \qquad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Admittance Parameters

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Hybrid Parameters



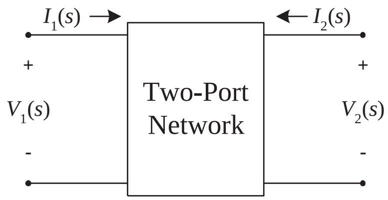
Hybrid Parameters

$$V_1 = h_{11}I_1 + h_{12}V_2 \qquad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Inverse Hybrid Parameters

$$I_1 = g_{11}V_1 + g_{12}I_2 \qquad \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

Transmission Parameters



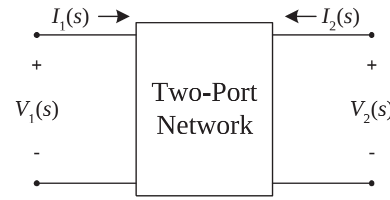
Transmission Parameters

$$V_1 = a_{11}V_2 - a_{12}I_2 \qquad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & b_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = A \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$
$$I_1 = a_{21}V_2 - a_{22}I_2$$

Inverse Transmission Parameters

$$V_2 = b_{11}V_1 - b_{12}I_1 \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} = B \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$
$$I_2 = b_{21}V_1 - b_{22}I_1$$

Transmission Parameter Conversion



- Altogether there are 6 sets of parameters
- Each set completely describes the two-port network
- Any set of parameters can be converted to any other set
- We have seen one example of a conversion
- A complete table of conversions is listed in the text (Pg. 933)
- You should have a copy of this in your notes for the final

Example 3: Two-Port Measurements

The following measurements were taken from a two-port network. Find the transmission parameters.

Port 2 Open

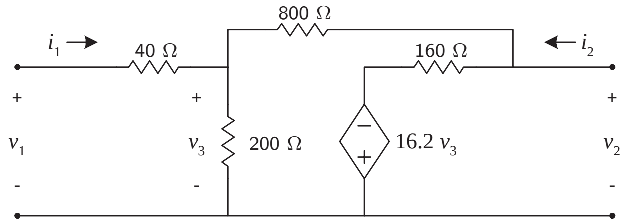
$$V_1 = 150\cos(4000t) \text{ V applied}$$
$$I_1 = 25\cos(4000t - 45^\circ) \text{ A measured}$$
$$V_2 = 1000\cos(4000t + 15^\circ) \text{ V measured}$$

Port 2 Shorted

$$V_1 = 30\cos(4000t) \text{ V applied}$$
$$I_1 = 1.5\cos(4000t + 30^\circ) \text{ A measured}$$
$$I_2 = 0.25\cos(4000t + 150^\circ) \text{ A measured}$$

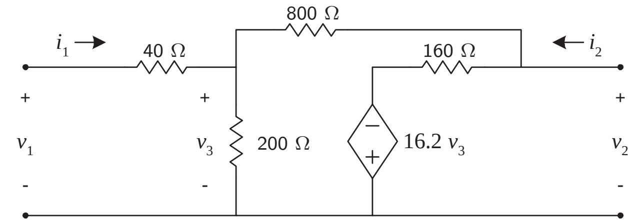
Example 3: Workspace

Example 4: Two-Port Analysis



Find the hybrid parameters for the circuit shown above.

Example 4: Workspace



Example 4: Workspace Continued

Example 5: Two-Port Measurements

The following measurements were taken from a two-port network. Find the transmission parameters.

Port 1 Open

$$V_1 = 1 \text{ mV}$$
$$V_2 = 10 \text{ V}$$
$$I_2 = 200\ \mu\text{A}$$

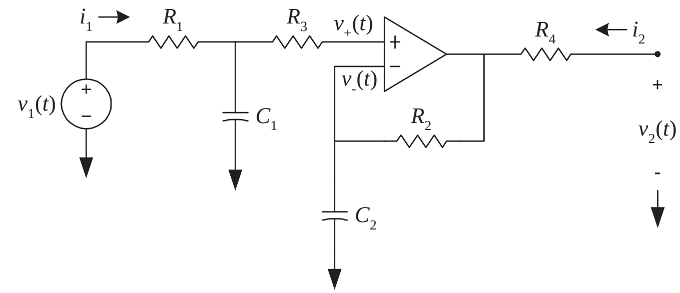
Port 1 Shorted

$$I_1 = -0.5\ \mu\text{A}$$
$$I_2 = 80\ \mu\text{A}$$
$$V_2 = 5 \text{ V}$$

Hint: $\triangle_b = b_{11}b_{22} - b_{12}b_{21}$, $a_{11} = \frac{b_{22}}{\triangle_b}$, $a_{12} = \frac{b_{12}}{\triangle_b}$, $a_{21} = \frac{b_{21}}{\triangle_b}$, and $a_{22} = \frac{b_{11}}{\triangle_b}$.

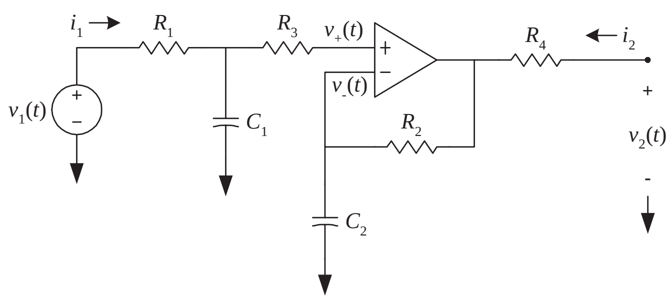
Example 5: Workspace

Example 6: Two-Port Analysis

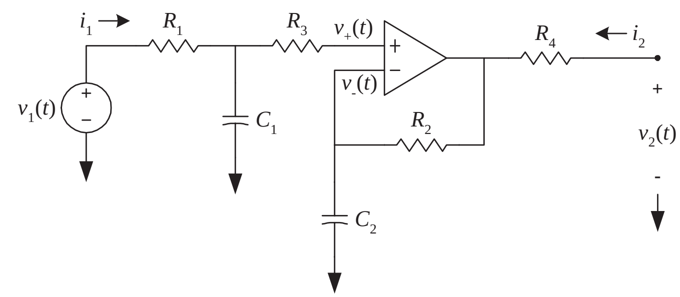


Find an expression for the transfer function, h_{11}, z_{11}, g_{12}, g_{22}, a_{11}, and y_{21}.

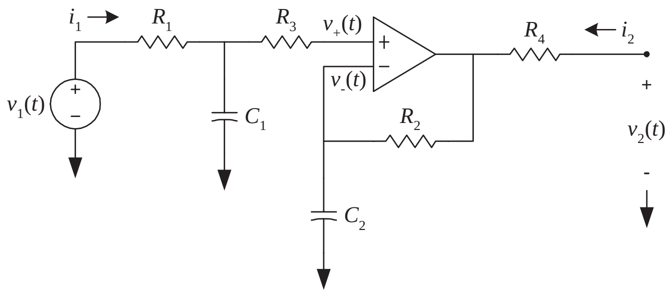
Example 6: Workspace



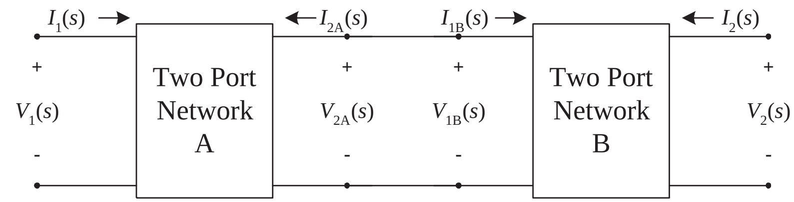
Example 6: Workspace Continued (1)



Example 6: Workspace Continued (2)

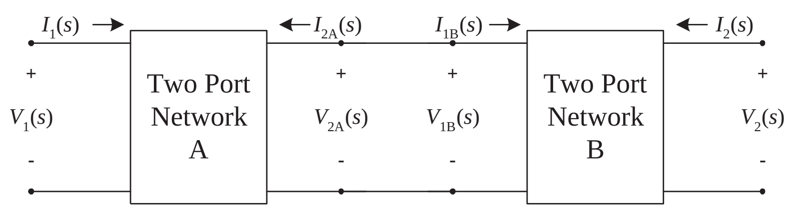


Cascaded Two-Port Networks



- Two networks are **cascaded** when the output of one is the input of the other
- Note that $V_{2A} = V_{1B}$ and $-I_{2A} = I_{1B}$
- The transmission parameters take advantage of these properties

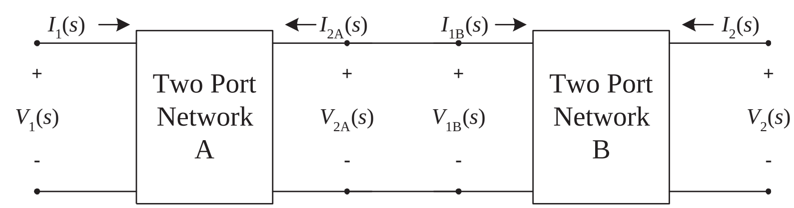
Cascaded Two-Port Networks



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_A \begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix} \qquad \begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_B \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix} = \begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_A \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_B \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Cascaded Two-Port Networks Continued

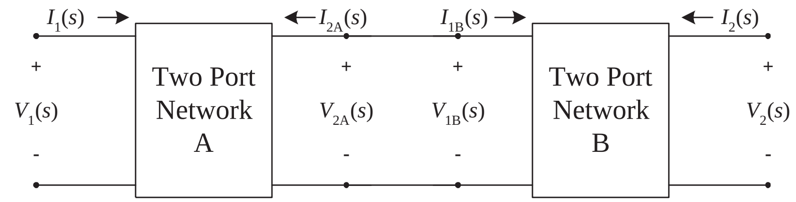


The inverse transmission parameters are also convenient for cascaded networks.

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_A \begin{bmatrix} V_{1B} \\ -I_{1B} \end{bmatrix} \qquad \begin{bmatrix} V_{2A} \\ I_{2A} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_B \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$\begin{bmatrix} V_{1B} \\ -I_{1B} \end{bmatrix} = \begin{bmatrix} V_{2A} \\ I_{2A} \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_A \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_B \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Cascaded Systems: Two-Port Networks versus $H(s)$



- Two-ports and transfer functions $H(s)$ are closely related
- $H(s)$ only relates the input signal to the output signal
- Two-ports relate both voltages and currents at each port
- You cannot cascade $H(s)$ unless the circuits are active
- Two-port networks have no such restriction
- Two-ports are used to design passive filters
- However, two-ports are more complicated than $H(s)$