Practical Analog Filters Overview

- Types of practical filters
- Filter specifications
- Tradeoffs
- Many examples
There are five ideal filters

- **Lowpass** filters pass low frequencies: $\omega < \omega_c$
- **Highpass** filters pass high frequencies: $\omega > \omega_c$
- **Bandpass** filters pass a range of frequencies: $\omega_{c1} < \omega < \omega_{c2}$
- **Bandstop** filters pass two ranges: $\omega < \omega_{c1}$ and $\omega > \omega_{c2}$
- **Notch** filters pass all frequencies except $\omega \approx \omega_c$
Ideal Filters Comments

- Phase is not shown
- $\omega_c$ is called the **cutoff frequency**
- Generally, the ideal phase is $0^\circ$ for all frequencies
- Can not build ideal filters in practice
- Real filters appear as rounded versions of ideal filters
- Most LTI systems can be thought of as non-ideal filters
Practical Filters

• Practical filters are usually designed to meet a set of specifications

• Lowpass and highpass filters usually have the following requirements
  – Passband range
  – Stopband range
  – Maximum ripple in the passband
  – Minimum attenuation in the stopband

• If we know the specifications, we can ask MATLAB to generate the filter for us

• There are four popular types of standard filters
  – Butterworth
  – Chebyshev Type I
  – Chebyshev Type II
  – Elliptic
Practical Filter Tradeoffs

- **Butterworth**
  - Highest order $H(s)$
  - No passband or stopband ripple

- **Chebyshev Type I**
  - No stopband ripple

- **Chebyshev Type II**
  - No passband ripple

- **Elliptic**
  - Lowest order $H(s)$
  - Passband and stopband ripple
Example 1: Lowpass Filter Specifications

Design a lowpass filter that meets the following specifications:

- The passband ripple is no more than 0.4455 dB
  \(0.95 \leq |H(j\omega)| \leq 1\)
- The stopband attenuation is at least 26.02 dB
  \(|H(j\omega)| \leq 0.05\)
- The passband ranges from 0–450 rad/s
- The stopband ranges from 550–\(\infty\) rad/s

Plot the magnitude of the resulting transfer function on a linear-linear plot, the Bode magnitude plot, the pole-zero plot, the impulse response, and the step response. Try the Butterworth, Chebyshev I, Chebyshev II, and elliptic filters.

The MATLAB code is posted on the course web site.
Example 2: Butterworth Lowpass

Butterworth Lowpass Filter Transfer Function  Order: 21

Frequency (rad/sec)
Example 2: Butterworth Lowpass

Butterworth Lowpass Filter Transfer Function  Order: 21

$|H(j\omega)|$ (dB)

$10^2$  $10^3$
Example 2: Butterworth Lowpass

Butterworth Lowpass Filter Pole–Zero Plot  Order: 21

Poles
Example 2: Butterworth Lowpass

Butterworth Lowpass Filter Impulse Response  Order: 21

![Butterworth Lowpass Filter Impulse Response](image)

**Order:** 21

**Time (s):** 0 to 0.25

**h(t):** -100 to 150
Example 2: Butterworth Lowpass

Butterworth Lowpass Filter Step Response  Order: 21

h(t)

Time (s)
Example 2: Chebyshev-I Lowpass

Chebyshev–I Lowpass Filter Transfer Function   Order: 8

|F(jω)|

Chebyshev-I Lowpass Filter Transfer Function  Order: 8

Frequency (rad/sec)
Example 2: Chebyshev-I Lowpass
Example 2: Chebyshev-I Lowpass

Chebyshev–I Lowpass Filter Pole–Zero Plot  Order:8

Poles
Example 2: Chebyshev-I Lowpass

Chebyshev–I Lowpass Filter Impulse Response   Order:8
Example 2: Chebyshev-I Lowpass

Chebyshev–I Lowpass Filter Step Response  Order: 8

Time (s)

h(t)
Example 2: Chebyshev-II Lowpass

Chebyshev–II Lowpass Filter Transfer Function  Order: 8

|\left|H(j\omega)\right| |
|---|
| 1  |

Frequency (rad/sec)

0 100 200 300 400 500 600 700 800 900 1000
Example 2: Chebyshev-II Lowpass
Example 2: Chebyshev-II Lowpass

Chebyshev–II Lowpass Filter Pole–Zero Plot   Order: 8

Poles
Zeros
Example 2: Chebyshev-II Lowpass

Chebyshev-II Lowpass Filter Impulse Response   Order:8
Example 2: Chebyshev-II Lowpass

Chebyshev-II Lowpass Filter Step Response  Order: 8
Example 2: Elliptic Lowpass

Elliptic Lowpass Filter Transfer Function  Order: 5
Example 2: Elliptic Lowpass

Elliptic Lowpass Filter Transfer Function   Order:5

$|H(j\omega)|$ (dB)

$10^2$ $10^3$
Example 2: Elliptic Lowpass

Elliptic Lowpass Filter Pole-Zero Plot  Order: 5

Poles
Zeros

Real Axis
Imaginary Axis
Example 2: Elliptic Lowpass

Elliptic Lowpass Filter Impulse Response  Order: 5
Example 2: Elliptic Lowpass

Elliptic Lowpass Filter Step Response  Order:5

![Elliptic Lowpass Filter Step Response](image-url)
Example 3: Highpass Filter Specifications

Design an elliptic highpass filter that meets the following specifications:

- The passband ripple is no more than 0.4455 dB
  \[0.95 \leq |H(j\omega)| \leq 1\]
- The stopband attenuation is at least 26.02 dB
  \[|H(j\omega)| \leq 0.05\]
- The passband ranges from 550–\(\infty\) rad/s
- The stopband ranges from 0–450 rad/s

Plot the magnitude of the resulting transfer function on a linear-linear plot, the Bode magnitude plot, the pole-zero plot, the impulse response, and the step response.
Example 3: Elliptic Highpass
Example 3: Elliptic Highpass

Elliptic Highpass Filter Transfer Function   Order:5
Example 3: Elliptic Highpass

Elliptic Highpass Filter Pole–Zero Plot

Order: 5

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Example 3: Elliptic Highpass

Elliptic Highpass Filter Impulse Response  Order:5
Example 3: Elliptic Highpass

Elliptic Highpass Filter Step Response  Order: 5
Example 3: Relevant MATLAB Code

\[
\begin{align*}
W_p &= 550; & \text{\% Passband ends} \\
W_s &= 450; & \text{\% Stopband begins} \\
R_p &= -20 \cdot \log_{10}(0.95); & \text{\% Maximum deviation from 1 in the passband (dB)} \\
R_s &= -20 \cdot \log_{10}(0.05); & \text{\% Minimum attenuation in the stopband (dB)} \\
[n,\omega_n] &= \text{ellipord}(W_p,W_s,R_p,R_s,'s'); \\
[B,A] &= \text{ellip}(n,R_p,R_s,\omega_n,'\text{high}', 's');
\end{align*}
\]
Example 4: Bandpass Filter Specifications

Design an Chebyshev type I bandpass filter that meets the following specifications:

- The passband ripple is no more than 0.4455 dB \((0.95 \leq |H(j\omega)| \leq 1)\)
- The stopband attenuation is at least 26.02 dB \(|H(j\omega)| \leq 0.05\)
- The passband ranges from 450–550 rad/s
- The stopband ranges from 0–400 rad/s and 500–∞ rad/s

Plot the magnitude of the resulting transfer function on a linear-linear plot, the Bode magnitude plot, the pole-zero plot, the impulse response, and the step response.
Example 4: Chebyshev-I Bandpass

Chebyshev–I Bandpass Filter Transfer Function   Order: 8
Example 4: Chebyshev-I Bandpass

Chebyshev–I Bandpass Filter Transfer Function  Order:8

|H(jω)| (dB)
Example 4: Chebyshev-I Bandpass

Chebyshev–I Bandpass Filter Pole–Zero Plot   Order:8

Poles
Zeros
Example 4: Chebyshev-I Bandpass

Chebyshev–I Bandpass Filter Impulse Response   Order:8

Time (s)
Example 4: Chebyshev-I Bandpass

Chebyshev–I Bandpass Filter Step Response  Order:8
Example 4: Relevant MATLAB Code

Wp = [450 550]; % Passband ends
Ws = [400 600]; % Stopband begins
Rp = -20*log10(0.95); % Maximum deviation from 1 in the passband (dB)
Rs = -20*log10(0.05); % Minimum attenuation in the stopband (dB)
[n,wn] = cheb1ord(Wp,Ws,Rp,Rs,'s');
[B,A] = cheby1(n,Rp,wn,'s');
Example 5: Bandstop Filter Specifications

Design an Chebyshev type II bandstop filter that meets the following specifications:

- The passband ripple is no more than 0.4455 dB \((0.95 \leq |H(j\omega)| \leq 1)\)
- The stopband attenuation is at least 26.02 dB \(|H(j\omega)| \leq 0.05\)
- The passband ranges from 0–400 rad/s and 500–∞ rad/s
- The stopband ranges from 450–550 rad/s

Plot the magnitude of the resulting transfer function on a linear-linear plot, the Bode magnitude plot, the pole-zero plot, the impulse response, and the step response.
Example 5: Chebyshev-II Bandstop

Chebyshev–II Bandstop Filter Transfer Function  Order: 8

|H(jω)|

Frequency (rad/sec)

0  100  200  300  400  500  600  700  800  900  1000
Example 5: Chebyshev-II Bandstop

Chebyshev–II Bandstop Filter Transfer Function   Order:8

|H(jω)| (dB)

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$10^2$  $10^3$
Example 5: Chebyshev-II Bandstop

Chebyshev–II Bandstop Filter Pole–Zero Plot  Order:8

- Imaginary Axis
- Real Axis
Example 5: Chebyshev-II Bandstop

Chebyshev–II Bandstop Filter Impulse Response  Order:8

Chebyshev-II Bandstop Filter Impulse Response   Order:8
Example 5: Chebyshev-II Bandstop

Chebyshev–II Bandstop Filter Step Response  Order:8
Example 5: Relevant MATLAB Code

Wp = [400 600];         % Passband ends  
Ws = [450 550];         % Stopband begins  
Rp = -20*log10(0.95);   % Maximum deviation from 1 in the passband (dB)  
Rs = -20*log10(0.05);   % Minimum attenuation in the stopband (dB)  
[n,wn] = cheb2ord(Wp,Ws,Rp,Rs,'s');  
[B,A] = cheby2(n,Rs,wn,'stop','s');
Practical Filters Summary

• In most applications, filter design begins with a set of specifications.

• We can use design tools like MATLAB to solve for the $H(s)$ that meets a set of frequency specifications.

• We can then use the cascade-form of transfer function synthesis.

• However, there are standard circuits for these types of filters that are less expensive and more tolerant of errors in component values.