Exam 1
January 26, 2005

ECE 222: Signals and Systems
Dr. McNames

- Write your full name above.
- Keep your exam flat during the entire exam.
- If you have to leave the exam temporarily, close the exam and leave it face down while you are out of the room.
- Turn off any cell phones or pagers that might interrupt the exam.
- Do not open the exam until instructed to do so.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- You will have 100 minutes to complete the exam.
- If you have extra time, double check your answers.
- Remember to include units with each of your answers.

Problem 1:_____ / 13
Problem 2:_____ / 10
Problem 3:_____ / 15
Problem 4:_____ / 12

Total:_____ / 50
1. Signals and Systems Concepts (13 pts)
Circle the appropriate answers to the multiple choice questions below. Note that some questions may have more than one correct answer that must be circled.

a. (3 pts) Which of the following signals have finite energy?

\[ x(t) = 1 \]
\[ x(t) = \cos(500t + \pi/3) \ u(t) \]
\[ x(t) = e^{2t} \ u(t) \]
\[ x(n) = \exp(-j\pi n) \ u[n] \]
\[ x[n] = 0 \]
\[ x(t) = e^{-5t} \ u(t) \]

b. (2 pts) Which of the following impulse responses represent causal systems? You may assume that each of the systems are linear and time-invariant.

\[ h(t) = 1 \]
\[ h(t) = \cos(500t + \pi/3) \ u(t) \]
\[ h(t) = e^{at} \ u(t), \ a > 0 \]
\[ h[n] = \exp(-j\pi n) \ u[n-1] \]
\[ h[n] = \exp(-j\pi n) \ u[n+1] \]
\[ h(t) = e^{at} \ u(t), \ a < 0 \]

c. (1 pt) What is the magnitude of the signal \( x(t) = e^{j\omega t} \)?

\[ |x(t)| = \] __________

d. (1 pt) What is the real part of the signal \( x(t) = e^{j\omega t} \)?

\[ \text{Re}\{x(t)\} = \] __________

e. (1 pt) What is the fundamental period of the signal \( x(t) = e^{j\omega t} \)?

\[ T_0 = \] __________

f. (1 pt) What is the value of the following expression?

\[ \int_{-\infty}^{\infty} e^{j5\pi t} \delta(t-1) \ dt = \] _________________

g. (1 pt) What is the maximum value of \( \delta[n] \)?

\[ \max_n \delta[n] = \] __________

h. (1 pt) What is the maximum value of \( \delta(t) \)?

\[ \max_t \delta[t] = \] __________

i. (1 pt) What properties must a system have in order for the output of a discrete-time system to be related to the input by the convolution sum?

Invertible  Memoryless  Causal  Stable  Linear  Time-Invariant

j. (1 pt) What properties must a system have in order for the output of a continuous-time system to be related to the input by the convolution integral?

Invertible  Memoryless  Causal  Stable  Linear  Time-Invariant
2. **Fundamentals of Signals (10 pts)**

Use the following signal to answer the questions below. You may assume that the signal is equal to zero outside of the time range shown.

![Signal Diagram](image)

a. (1 pt) What is the signal energy of \( x[n] \)?

\[ E_x = \]

b. (2 pts) Draw \( x[2 - n] \) below.

![Graph of x[2 - n]](image)

c. (2 pts) Draw the odd component of \( x[n] \) below.

![Graph of odd component] (image)
2. Fundamentals of Signals Continued (10 pts)

Use the following signal to answer the questions below. You may assume that the signal is equal to zero outside of the time range shown.

\[ x(t) \]

\[ -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]

\[ x(t) \]

\[ 1 \quad 2 \]

\[ t \]

\[ -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]

\[ 1 \quad 2 \]

\[ t \]

d. (1 pt) What is the signal power of \( x(t) \)?

\[ P_\infty = \]

e. (2 pts) Draw the \( x(1 - 2t) \) below.

\[ \]

\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

\[ -1 \quad -2 \]

\[ -3 \quad -4 \]

\[ t \]

\[ -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]

\[ \]

\[ 1 \quad 2 \]

\[ t \]

f. (2 pts) Draw the even component \( x(t) \) below.

\[ \]

\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

\[ -1 \quad -2 \]

\[ -3 \quad -4 \]

\[ t \]

\[ -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]

\[ \]
3. Properties of Systems (15 pts)

Fill each cell of the table with a Y if the system has the corresponding property and N if the system does not have the property. The continuous-time system has an input signal \( x(t) \) and each discrete-time system has an input signal \( x[n] \).

<table>
<thead>
<tr>
<th>System</th>
<th>Memoryless</th>
<th>Invertible</th>
<th>Causal</th>
<th>Stable</th>
<th>Time Invariant</th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y[n] = x[-3] )</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>( y(t) = \int_{t-25}^{t}</td>
<td>x(\tau)</td>
<td>^2 , d\tau )</td>
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<tr>
<td>( y[n] = x[n] - x[n-1] + x[n-2] - 2n )</td>
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<tr>
<td>( y(t) = x(t) \cdot e^{j\omega t} )</td>
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<tr>
<td>( y(t) = 5 \frac{dx(t-2)}{dt} )</td>
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</tbody>
</table>
4. Convolution Sum (12 pts)
Consider a linear time-invariant discrete-time system with the input signal $x[n]$ and impulse response $h[n]$ shown below for the questions that follow. You may assume that both signals are equal to zero outside of the time range shown.

\[ x[n] \]
\[ h[n] \]

a. (1 pt) Is the system memoryless? (Circle one)
   Yes    No

b. (1 pt) Is the system stable? (Circle one)
   Yes    No

c. (1 pt) Is the system causal? (Circle one)
   Yes    No

d. (1 pt) What is the smallest sample time $n_0$ such that $y[n] = 0$ for all $n \geq n_0$?
   $n_0 = $

e. (1 pt) What is the largest sample time $n_0$ such that $y[n] = 0$ for all $n \leq n_0$?
   $n_0 = $

f. (1 pt) If the input signal is bounded such that $|x[n]| \leq 7$, what is the maximum possible output that the system could produce? If the maximum is unbounded, write $\infty$.
   $\max y[n] = $

g. (1 pt) Draw the discrete-time signal $h[n-k]$ versus $k$ for $n = -3$ on the axis given below.

h. (1 pt) What is the output of the system for $n = -3$?
   $y[-3] =$
4. Convolution Sum Continued (12 pts)
The input signal $x[n]$ and impulse response $h[n]$ are repeated below from the previous page.

![Graph of x[n] and h[n]](image)

i. (1 pt) Draw the discrete-time signal $h[n-k]$ versus $k$ for $n=1$ on the axis given below.

![Graph of h[n-k] for n=1]](image)

j. (1 pt) What is the output of the system for $n=1$?

$y[1] = \ldots$

k. (1 pt) Draw the discrete-time signal $h[n-k]$ versus $k$ for $n=4$ on the axis given below.

![Graph of h[n-k] for n=4]](image)

l. (1 pt) What is the output of the system for $n=4$?

$y[4] = \ldots$