Exam 1A Solutions
January 29, 2003

ECE 222: Signals and Systems
Dr. McNames

- Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
- Do not open the exam until instructed to do so.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
- You will have 100 minutes to complete the exam.
- If you have extra time, double check your answers.
- Remember to include units with each of your answers.
- You are not allowed to use a calculator during this exam.

Problem 1:______ / 10
Problem 2:______ / 15
Problem 3:______ / 12
Problem 4:______ / 13

Total:______ / 50

First Letter in Last Name:______________
6-Digit Identification Number:_____________
Student Identification Number:_____________
1. Fundamentals of Signals (10 pts)
Use the following signal to answer the questions below. You may assume that the signal is equal to zero outside of the time range shown.

\[ x[n] \]

a. (1 pt) What is the signal energy of \( x[n] \)?
\[ E_x = 4 + 4 + 4 + 4 + 1 + 1 = 18 \]
b. (2 pts) Draw \( 0.5 \cdot x[n + 2] \) below.

c. (2 pts) Draw the even component of \( x[n] \) below.
1. Fundamentals of Signals Continued (10 pts)
Use the following signal to answer the questions below. You may assume that the signal is equal to zero outside of the time range shown.

\[
x(t)
\]

\[
-4 -3 -2 -1 0 1 2 3 4
\]

\[
-2 -1 1 2
\]

\[
t
\]

\[
E_\infty = 4 + 8 = 12
\]

d. (1 pt) What is the signal energy of \(x(t)\)?

\[
E_\infty = 4 + 8 = 12
\]

e. (2 pts) Draw the \(-x(-t-1)\) below.

\[
-7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8
\]

\[
-2 -1 1 2
\]

\[
t
\]

e. (2 pts) Draw the odd component \(x(t)\) below.
2. Properties of Systems (15 pts)

Fill each cell of the table with a Y if the system has the corresponding property and N if the system does not have the property. The continuous-time system has an input signal $x(t)$ and each discrete-time system has an input signal $x[n]$.

<table>
<thead>
<tr>
<th>System</th>
<th>Memoryless</th>
<th>Invertible</th>
<th>Causal</th>
<th>Stable</th>
<th>Time Invariant</th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(t) = \int_{t-5}^{t+5}</td>
<td>x(\tau)</td>
<td>, d\tau$</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$y[n] = x[n] + 2$</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>$y(t) = \frac{d^2 x(t+3)}{dt^2}$</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$y[n] = x[5n - 3]$</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$y(t) = \ln</td>
<td>t</td>
<td>\cdot x(t)$</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
</tbody>
</table>

The last system is the most difficult because at $t=0$, $\ln |t| = -\infty$. 
3. Convolution Sum (12 pts)
Consider a linear time-invariant discrete-time system with the input signal $x[n]$ and impulse response $h[n]$ shown below for the questions that follow. You may assume that both signals are equal to zero outside of the time range shown.

![Graph](image)

a. (1 pt) Is the system memoryless? (Circle one)
   - Yes
   - No

b. (1 pt) Is the system stable? (Circle one)
   - Yes
   - No

c. (1 pt) Is the system causal? (Circle one)
   - Yes
   - No

d. (1 pt) What is the output of the system for $n = -4$?
   $y[-4] = -2$

e. (1 pt) What is the smallest sample time $n_0$ such that $y[n] = 0$ for all $n \geq n_0$?
   $n_0 = 9$

f. (1 pt) If the input signal is bounded such that $|x[n]| \leq 2$, what is the maximum possible output that the system could produce? If the maximum is unbounded, write $\infty$.
   $\max y[n] = 2 + 4 + 2 + 2 = 14$

g. (1 pt) Draw the discrete-time signal $h[n-k]$ versus $k$ for $n = -3$ on the axis given below.

![Graph](image)

h. (1 pt) What is the output of the system for $n = -3$?
   $y[-3] = 4$
3. Convolution Sum Continued (12 pts)
The input signal \( x[n] \) and impulse response \( h[n] \) are repeated below from the previous page.

\[ x[n] \quad h[n] \]

\[ \begin{array}{cccccccc}
-4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
2 & 1 & -1 & -2 & 1 & 2 & 1 & 2 \\
\end{array} \]

i. (1 pt) Draw the discrete-time signal \( h[n-k] \) versus \( k \) for \( n = 1 \) on the axis given below.

\[ k \]

\[ \begin{array}{cccccccc}
-11 & -10 & -9 & -8 & -7 & -6 & -5 & -4 \\
2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array} \]

j. (1 pt) What is the output of the system for \( n = 1 \)?

\[ y[1] = 2 + 4 + 2 = 8 \]

k. (1 pt) Draw the discrete-time signal \( h[n-k] \) versus \( k \) for \( n = 4 \) on the axis given below.

\[ k \]

\[ \begin{array}{cccccccc}
-11 & -10 & -9 & -8 & -7 & -6 & -5 & -4 \\
2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array} \]

l. (1 pt) What is the output of the system for \( n = 4 \)?

\[ y[4] = 4 + 2 = 6 \]
4. Convolution Integral (13 pts)
Consider a linear time-invariant system with the impulse response \( h(t) \) and input \( x(t) \) as shown below. Plot \( x(t - \tau) \) and \( h(\tau) \) for each of the intervals specified below. Label \( t+2, t+1, t, \) and \( t-1 \) on the plots. If the system output is zero for the interval write \( y(t) = 0 \). If it is not zero, write the expression for the convolution integral (the expression may contain more than one integral).

\[
x(t) \quad h(t) \quad y(t)
\]

\[
-2 \quad -1 \quad 1 \quad 2 \quad \tau
\]

\[
x(t) \quad h(t)
\]

\[
-2 \quad -1 \quad -1 \quad 1 \quad 2 \quad 3 \quad t
\]

\[
y(t) = 0
\]

a. (2 pts) Plot \( x(t - \tau) \) \& \( h(\tau) \) and write the expression for the convolution integral(s) for \( -2 \leq t \leq -1 \). Do \underline{not} evaluate the integral(s).

\[
y(t) = 0
\]

b. (3 pts) Plot \( x(t - \tau) \) \& \( h(\tau) \) and write the expression for the convolution integral(s) for \( -1 \leq t \leq 0 \). Do \underline{not} evaluate the integral(s).

\[
y(t) = \int_{1}^{t+2} -\left(\tau - (t + 2)\right) \frac{1}{2} (\tau - 1) \, d\tau
\]
4. Convolution Integral Continued (13 pts)

c. (5 pts) Plot \( x(t - \tau) \) & \( h(\tau) \) and write the expression for the convolution integral(s) for \( 1 \leq t \leq 2 \). Do not evaluate the integral(s).

\[ y(t) = \int_{1}^{t} -\frac{1}{2}(\tau - 1) \, d\tau + \int_{t+1}^{3} -\left(\tau - (t + 2)\right)\frac{1}{2}(\tau - 1) \, d\tau \]


d. (3 pts) Plot \( x(t - \tau) \) & \( h(\tau) \) and write the expression for the convolution integral(s) for \( 3 \leq t \leq 4 \). Do not evaluate the integral(s).

\[ y(t) = \int_{t-1}^{3} -\frac{1}{2}(\tau - 1) \, d\tau \]