Exam 1
February 4, 2002

ECE 222: Signals and Systems
Dr. McNames

• Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
• You are not allowed to use a calculator on this exam.
• Do not begin the exam until instructed to do so.
• You have 100 minutes to complete the exam.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages.

Problem 1:_____ / 8
Problem 2:_____ / 10
Problem 3:_____ / 15
Problem 4:_____ / 13
Problem 5:_____ / 14

Total:_____ / 60

First Letter in Last Name:_____________
6-Digit Identification Number:_____________
Student Identification Number:_____________
1. Fundamental Concepts (8 pts)
Circle True or False for each of the questions below.

a. (1 pt) Exponential and sinusoidal signals arise naturally in the analysis of linear systems.
   True   False

b. (1 pt) A decaying exponential function, $e^{-at}$, has even symmetry.
   True   False

c. (1 pt) A decaying exponential function, $e^{-at}$, has odd symmetry.
   True   False

d. (1 pt) Signals of finite amplitude and infinite duration, such as the unit step function, have finite energy.
   True   False

e. (1 pt) The impulse response of a system is all that is needed to calculate the output if the system is time invariant.
   True   False

f. (1 pt) All circuits that contain resistors, capacitors, inductors, transformers, and ideal op amps are linear.
   True   False

g. (1 pt) All circuits that contain resistors, capacitors, inductors, transformers, and ideal op amps are time-invariant.
   True   False

h. (1 pt) It is not physically possible to build a non-causal circuit.
   True   False
2. Fundamentals of Signals (10 pts)
Use the following signals to answer the questions below. You may assume that both signals are equal to zero outside of the time range shown.

a. (1 pt) What is the signal energy of $x[n]$?
   
   $E_\infty =$

b. (1 pt) What is the average signal power of $x[n]$?
   
   $P_\infty =$

c. (1 pt) What type of symmetry does $x[n]$ have? Circle one of the options below.

   Odd   Even   None

d. (2 pts) Draw $0.5 \cdot x[n + 2]$ below.

[Diagram of signal $x[n]$]

[Diagram of signal $x(t)$]

e. (1 pt) What is the signal energy of $x(t)$?
   
   $E_\infty =$

f. (1 pt) What is the average signal power of $x(t)$?
   
   $P_\infty =$

g. (1 pt) What type of symmetry does $x(t)$ have? Circle one of the options below.

   Odd   Even   None

h. (2 pts) Draw $x \left( \frac{1}{2} (t - 1) \right)$ below.

[Diagram of signal $x \left( \frac{1}{2} (t - 1) \right)$]
3. Properties of Systems (15 pts)
Fill each cell of the table with a Y if the system has the corresponding property and N if the system does not have the property. The continuous-time system has an input signal \( x(t) \) and each discrete-time system has an input signal \( x[n] \).

<table>
<thead>
<tr>
<th>System</th>
<th>Memoryless</th>
<th>Invertible</th>
<th>Causal</th>
<th>Stable</th>
<th>Time Invariant</th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(t) = x(t)^3 )</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>( y[n] = x[n^3] )</td>
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<tr>
<td>( y(t) = \int_{-\infty}^{0} x(\tau) , d\tau )</td>
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<tr>
<td>( y[n] = \sum_{k=n-10}^{n+5} x[k-5] )</td>
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<tr>
<td>( y(t) = x(t) \cdot (\sin(2\pi t) + 2) )</td>
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</tbody>
</table>
4. **Convolution Sum (13 pts)**
Consider a linear time-invariant discrete-time system with the input signal $x[n]$ and impulse response $h[n]$ shown below for the questions that follow. You may assume that both signals are equal to zero outside of the time range shown.

![System Diagram]

a. (1 pt) Is the system causal? (Circle one)
   Yes  No

b. (1 pt) Is the system memoryless? (Circle one)
   Yes  No

c. (1 pt) Is the system stable? (Circle one)
   Yes  No

d. (1 pt) What is the output of the system for $n = -4$?
   $y[-4] = \quad$

e. (2 pts) Draw the discrete-time signal $h[n-k]$ versus $k$ for $n = -3$ on the axis given below.

![Graph]

f. (1 pts) What is the output of the system for $n = -3$?
   $y[-3] = \quad$
4. Convolution Sum Continued (13 pts)
The input signal $x[n]$ and impulse response $h[n]$ are repeated below from the previous page for your convenience.

\[ x[n] \]

\begin{align*}
&-4 & -3 & -2 & -1 & 1 & 2 & 3 & 4 & 5 & 6 \\
&-2 & -1 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
&1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
&-1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\
\end{align*}

\[ h[n] \]

\begin{align*}
&-4 & -3 & -2 & -1 & 1 & 2 & 3 & 4 & 5 & 6 \\
&-2 & -1 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
&1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
&-1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 & -10 \\
\end{align*}

g. (2 pts) Draw the discrete-time signal $h[n-k]$ versus $k$ for $n = 0$ on the axis given below.

h. (1 pts) What is the output of the system for $n = 0$?
$y[0] =$

i. (2 pts) Draw the discrete-time signal $h[n-k]$ versus $k$ for $n = 2$ on the axis given below.

j. (1 pts) What is the output of the system for $n = 2$?
$y[2] =$
5. Convolution Integral (14 pts)
Consider a linear time-invariant system with the impulse response \( h(t) \) and input \( x(t) \) as shown below. Plot \( x(t - \tau) \) and \( h(\tau) \) for each of the intervals specified below. Label \( t + 1, t, t - 1, \) and \( t - 2 \) on the plots. If the system output is zero for the interval write \( y(t) = 0 \). If it is not zero, write the expression for the convolution integral (the expression may contain more than one integral).

a. (2 pts) Plot \( x(t - \tau) \) & \( h(\tau) \) and write the expression for the convolution integral(s) for \( t \leq -1 \). Do not evaluate the integral(s).

\[
y(t) =
\]

b. (3 pts) Plot \( x(t - \tau) \) & \( h(\tau) \) and write the expression for the convolution integral(s) for \( -1 \leq t \leq 0 \). Do not evaluate the integral(s).

\[
y(t) =
\]
5. Convolution Continued (14 pts)

c. (7 pts) Plot \( x(t - \tau) \) & \( h(\tau) \) and write the expression for the convolution integral(s) for \( 1 \leq t < 2 \). Do not evaluate the integral(s).

\[
\begin{align*}
\tau & = \begin{array}{cccccccc}
4 & -3 & -2 & -1 & 1 & 2 & 3 & 4 \\
\tau & = \begin{array}{cccccccc}
4 & -3 & -2 & -1 & 1 & 2 & 3 & 4 \\
\end{array}
\end{align*}
\]

Let \( y(t) = \) 

\[
\begin{align*}
\end{align*}
\]

d. (2 pts) Plot \( x(t - \tau) \) & \( h(\tau) \) and write the expression for the convolution integral(s) for \( 4 \leq t \). Do not evaluate the integral(s).

\[
\begin{align*}
\tau & = \begin{array}{cccccccc}
4 & -3 & -2 & -1 & 1 & 2 & 3 & 4 \\
\tau & = \begin{array}{cccccccc}
4 & -3 & -2 & -1 & 1 & 2 & 3 & 4 \\
\end{array}
\end{align*}
\]

Let \( y(t) = \) 

\[
\begin{align*}
\end{align*}
\]