Exam 1 Solutions
April 25, 2001

ECE 222: Signals and Systems
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- Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
- You are not allowed to use a calculator on this exam.
- Do not begin the exam until instructed to do so.
- You have 100 minutes to complete the exam.
- Once you begin, write your student ID at the top of each page and make sure you have all the pages.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages.

Problem 1:______ /  7
Problem 2:______ /  13
Problem 3:______ /  16
Problem 4:______ /  9

Total:______ /  45

First Letter in Last Name:_____________

6-Digit Identification Number:_____________

Student Identification Number:_____________
1. Basis Functions (7 Points)
Use the following transfer function to answer the questions below.

\[ H(s) = \frac{3s^3 - 4s^2 + 6s + 12}{s^4 - 12s^3 + 2s^2 + 4s} \]

a. (5 pts) Draw the block diagram for a circuit that implements this transfer function. Use the symbol \( \frac{1}{s} \) for integrators, \( \Sigma \) for adders, and \( a_n \) for multipliers (amplifiers).

![Block Diagram](image.png)

b. (1 pt) Find the initial value of system output due to a unit step input \( u(t) \).

\[ \lim_{t \to 0^+} y(t) = \lim_{s \to \infty} sH(s) = 0 \]

\[ \frac{1}{s} \]

\[ H(s) = 0 \]

c. (1 pt) Find the final value of system output due to a unit step input \( u(t) \).

\[ \lim_{t \to \infty} y(t) = \lim_{s \to 0} sH(s) = +\infty \]

\[ \frac{1}{s} \]

\[ H(s) = +\infty \]
2. Inverse Laplace Transforms (13 points)
Find an expression for the inverse Laplace transforms of the following functions.

a. (3 pts) \[ X(s) = \frac{5}{s^2} + \frac{6}{s^2 + 9} + \frac{10e^{-2s}}{s} \]

\[ x(t) = 5r(t) + 2\sin(3t)u(t) + 10u(t - 2) \]

b. (5 pts) \[ X(s) = \frac{5}{s + 1} + \frac{1 - j}{s + 2 - j3} + \frac{1 + j}{s + 2 + j3} \]

\[ x(t) = \left[ 5e^{-t} + 2\sqrt{2}e^{-2t}\cos(3t - 45^\circ) \right] u(t) = \left[ 5e^{-t} + 2e^{-2t}\cos(3t) + 2e^{-2t}\sin(3t) \right] u(t) \]

c. (5 pts) \[ X(s) = \frac{12}{(s + 4)(s + 2)^2} \]

\[ \frac{12}{(s + 4)(s + 2)^2} = \frac{k_1}{s + 4} + \frac{k_2}{s + 2} + \frac{k_3}{(s + 2)^2} \]

\[ 12 = k_1(s^2 + 4s + 4) + k_2(s^2 + 6s + 8) + k_3(s + 4) \]

\[ s^2 : 0 = k_1 + k_2; k_1 = -k_2 \]

\[ s^1 : 0 = 4k_1 + 6k_2 + k_3 = 2k_2 + k_3; k_3 = -2k_2 \]

\[ s^0 : 12 = 4k_1 + 8k_2 + 4k_3 = -4k_2 \]

\[ k_2 = -3; k_3 = 6; k_1 = 3 \]

\[ x(t) = \left[ 3e^{-4t} - 3e^{-2t} + 6te^{-2t} \right] u(t) \]
3. Laplace Circuit Analysis (16 pts)

d. (8 pts) Draw the s-domain equivalent circuit. Use the s-domain equivalents for the capacitor and inductor that include a current source, if appropriate. If you cannot solve for initial conditions, make up values and note this in your solution. Hint: recall that a voltage source with a value of 0 V is equivalent to a short circuit.

\[ 2 \text{ A} \quad 2 \text{ e}^{-t} \text{ u}(t) \text{ V} \]

\[ 0.5 \text{ F} \quad 3 \text{ H} \quad 5 \Omega \quad 4 \Omega \]

\[ \frac{12}{2s} \quad \frac{2}{s} \quad \frac{2}{s} \quad \frac{2}{(s+1)} \]

\[ 2/5 + \frac{V_1 - \frac{2}{s+1}}{2s} - 5 - \frac{2}{s} = 0 \]

\[ \frac{V_2}{4} + \frac{2}{s} - \frac{2}{s} + \frac{V_2 - \frac{2}{s+1}}{3s} = 0 \]

e. (8 pts) Use nodal analysis to write two independent equations in terms of the nodal voltages \( V_1 \) and \( V_2 \) in the s-domain. You do not need to simplify your equations.
4. Op Amps & Initial/Final Value Theorems (9 pts)
Assume zero initial conditions for this problem.

a. (3 pts) Find the transfer function \( H(s) \) for the circuit below. Simplify your expression as much as possible.

\[
H(s) = -\frac{Z_B}{Z_A} = -\frac{1 + R_B}{C_B s + 1} = -\frac{1}{C_A s} \left( R_B C_B s + 1 \right)
\]

b. (4 pts) Find a the transfer function \( H(s) \) for the circuit below. Simplify your expression as much as possible.

\[
V_s = \frac{R_a}{sL + R_1} V_i = \frac{1}{C} \frac{R_1}{R_2 + \frac{1}{C}} V_o
\]

\[
H(s) = \frac{V_o}{V_s} = \frac{R_1}{sL + R_1} \frac{R_2 + \frac{1}{C}}{R_2} = \frac{R_1 (R_2 C + 1)}{sL + R_1} = \left( \frac{R_1 R_2 C}{L} \right) s + \frac{R_1 C}{s + \frac{1}{C}}
\]

c. (1 pt) Find the final value of \( v_o(t) \) due to a unit impulse input voltage, \( v_i(t) = \delta(t) \), for the circuit in part a.

\[
v_o(\infty) = \lim_{t \to \infty} sV_o(s) = \lim_{s \to 0} sH(s) \times 1 = \lim_{s \to 0} s \left( \frac{C_A}{C_B} \right) \frac{R_B C_B + 1}{sL + R_1} = 0
\]

d. (1 pt) Find the initial value of \( v_o(t) \) due to a unit impulse input voltage, \( v_i(t) = \delta(t) \), for the circuit in part b.

\[
v_o(0+) = \lim_{s \to \infty} sV_o(s) = \lim_{s \to \infty} sH(s) \times 1 = \lim_{s \to \infty} s \frac{R_1 (R_2 C + 1)}{sL + R_1} = +\infty
\]