Exam 1
January 31, 2000

ECE 222: Signals and Systems
Dr. McNames

• Write your 6-digit identification number and student identification numbers below.
• Do not begin the exam or look at the problems until instructed to do so.
• You have 100 minutes to complete the exam.
• Once you begin, write your student ID at the top of each page and make sure you have all 7 pages.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages.
• NOTE: The book has an error in the Laplace transform tables. The Laplace transform of \( f(t-a)u(t-a) = e^{-as}F(s) \) for \( a > 0 \). The sign of the exponential should be negative, as shown here.

Problem 1:______ / 14
Problem 2:______ / 15
Problem 3:______ / 15
Problem 4:______ / 15
Problem 5:______ / 16

Total:______ / 75

6-Digit Identification Number:_____________
Student Identification Number:_____________
1. Basis Functions (14 Points)

![Graph of y(t)](image)

a. (5 pts) Write an expression for y(t) shown above using the basis functions $\delta(t)$, $u(t)$, and $r(t)$.

$$y(t) =$$

b. (3 pts) Write the Laplace transform of y(t) using the expression found in part a.

$$Y(s) =$$

c. (3 pts) Write an expression for the derivative of y(t) using the basis functions.

$$\frac{dy(t)}{dt} =$$

d. (3 pts) Write an expression for the integral of y(t) using the basis functions.

$$\int_{-\infty}^{t} y(\tau) d\tau =$$
2. **Inverse Laplace Transforms (15 points)**

Find \( f(t) \) for the following functions.

a. (5 pts) \( F(s) = \frac{16s^2 + 180s + 386}{(s + 2)(s + 5)(s + 7)} \)

\[ f(t) = \]

b. (5 pts) \( F(s) = \frac{9s^2 + 32s + 9}{s(s + 3)^2} \)

\[ f(t) = \]

c. (5 pts) \( F(s) = \frac{-6s^2 + 16s - 30}{s(s^2 + 4s + 5)} \)

\[ f(t) = \]
3. Laplace Circuit Analysis (15 pts)
Draw the s-domain equivalent circuits.

a. (7 pts) Use the s-domain equivalents for the capacitor and inductor that include a s-domain current source, if appropriate.

![Equivalent Circuit diagram](image)

b. (8 pts) Use the s-domain equivalents for the capacitor and inductor that include a s-domain voltage source, if appropriate.

![Equivalent Circuit diagram](image)
4. Transfer Functions & Initial/Final Value Theorems (15 pts)
Find the transfer functions $H(s) = \frac{V_o(s)}{V_i(s)}$ for the circuits below. Write your answers as a ratio of two polynomials in $s$ and simplify your expressions as much as possible.

a. (7 pts)

![Circuit Diagram](image)

$H(s) =$

b. (6 pts)

![Circuit Diagram](image)

$H(s) =$

c. (1 pt) Find the final value of $V_o(t)$ due to a unit impulse input voltage, $V_i(t) = \delta(t)$, for the circuit in part a.

$V_o(\infty) =$

d. (1 pt) Find the initial value of $V_o(t)$ due to a unit impulse input voltage, $V_i(t) = \delta(t)$, for the circuit in part a.

$V_o(0+) =$
5. Convolution (16 pts)
Consider a linear time-invariant system with the impulse response \( h(t) = e^{-t}u(t) \) for the questions below.

\[
\begin{array}{c}
x(t) \\
\downarrow \\
H(s) \\
\downarrow \\
y(t)
\end{array}
\]

a. (1 pt) Find the output of the system when the input is a unit impulse, \( x(t) = \delta(t) \).

\[ y(t) = \]

b. (4 pts) Find the output of the system when the input is a unit step, \( x(t) = u(t) \).

\[ y(t) = \]

c. (5 pts) Find the output of the system when the input is a unit ramp, \( x(t) = r(t) \).

\[ y(t) = \]
5. Convolution Continued

d. (6 pts) Find the output of the system when the input is the waveform shown below. Hint: you may use your answers found in parts a. through c to help answer this question.

\[ y(t) = \begin{cases} 
    t \leq 0 \\
    0 \leq t \leq 1 \\
    1 \leq t \leq 2 \\
    2 \leq t \leq 3 \\
    3 \leq t \leq 4 \\
    4 \leq t 
\end{cases} \]