Final Exam
June 6, 2000

ECE 222: Signals and Systems
Dr. McNames

- You are not allowed to use calculators on this exam.
- Write your 6-digit identification number and student identification numbers below.
- Do not begin the exam or look at the problems until instructed to do so.
- You have 100 minutes to complete the exam.
- Once you begin, write your student ID at the top of each page and make sure you have all the pages.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages.
- If you have extra time, double check your answers. If you run out of time, write the relevant equations that can be used to help solve the problem and a note describing your approach.

Problem 1:______ / 10
Problem 2:______ / 20
Problem 3:______ / 12
Problem 4:______ / 18

Total:______ / 60

6-Digit Identification Number:_____________

Student Identification Number:_____________
1. Basis Functions (10 Points)

a. (3 pts) Write an expression for \( y(t) \) shown above using the basis functions \( \delta(t) \), \( u(t) \), and \( r(t) \).

\[ y(t) = \]

b. (3 pts) Write the Laplace transform of \( y(t) \) using the expression found in part a.

\[ Y(s) = \]

c. (2 pts) Write an expression for the derivative of \( y(t) \) using the basis functions.

\[ \frac{dy(t)}{dt} = \]

d. (2 pts) Write an expression for the integral of \( y(t) \) using the basis functions.

\[ \int_{-\infty}^{t} y(\tau)d\tau = \]
2. Laplace Transform Circuit Analysis (20 Points)

![Circuit Diagram]

a. (3 pts) Find the transfer function \( H(s) = \frac{V_o(s)}{V_g(s)} \) for the circuit above in terms of the symbolic impedances \( Z_1, Z_2, \) and \( Z_3. \) Simplify your expression as much as possible.

\[
H(s) = \ldots
\]

b. (4 pts) Use your answer from part a. to find the transfer function for the circuit below, \( H(s) = \frac{V_o(s)}{V_g(s)} \), in terms of \( C, L, \) and \( R. \) Write you answer as a ratio of two polynomials in \( s. \) Simplify the expression such that the highest power of \( s \) in the denominator has a coefficient of 1.

![Circuit Diagram]

\[
H(s) = \ldots
\]
2. Laplace Transform Circuit Analysis – Continued

c. (4 pts) What values of \( R \), and \( L \) make the denominator equal to \((s + 5)(s + 20)\) if \( C = 4 \) mF?

\[
R = \quad L =
\]

d. (4 pts) Using the values found in part c., what is \( v_o(t) \) if \( v_g(t) = 30r(t) \)?

\[
v_o(t) =
\]

e. (1 pts) Using the values found in part c., what is the initial value of \( v_o(t) \) if \( v_g(t) = 10u(t) \)?

\[
\lim_{t \to 0} v_o(t) =
\]

f. (1 pts) Using the values found in part c., what is the final value of \( v_o(t) \) if \( v_g(t) = 10u(t) \)?

\[
\lim_{t \to \infty} v_o(t) =
\]

g. (1 pt) Using the values found in part c., what is \( H(j\omega) \) as \( \omega \) approaches zero (DC)?

\[
\lim_{\omega \to 0} H(j\omega) =
\]

h. (1 pt) Using the values found in part c., what is \( H(j\omega) \) as \( \omega \) approaches infinity (high frequencies)?

\[
\lim_{\omega \to \infty} H(j\omega) =
\]

i. (1 pt) What type of filter is this?
3. Two Port Networks (12 pts)

The following DC measurements were made on the resistive 2-port network.

<table>
<thead>
<tr>
<th>Measurement 1</th>
<th>Measurement 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1 = 10 \text{ V}$</td>
<td>$V_1 = 0 \text{ V}$</td>
</tr>
<tr>
<td>$I_1 = 250 \text{ mA}$</td>
<td>$I_1 = -30 \text{ mA}$</td>
</tr>
<tr>
<td>$V_2 = 0 \text{ V}$</td>
<td>$V_2 = 6 \text{ V}$</td>
</tr>
<tr>
<td>$I_2 = -30 \text{ mA}$</td>
<td>$I_2 = 6 \text{ mA}$</td>
</tr>
</tbody>
</table>

a. (4 pts) What are the $y$ parameters?

$$y_{11} = \quad y_{12} = \quad y_{21} = \quad y_{22} =$$

b. (4 pts) What are the $g$ parameters?

$$g_{11} = \quad g_{12} = \quad g_{21} = \quad g_{22} =$$

c. (4 pts) Find the parameters $R_1$, $\alpha$, $R_2$, and $\beta$ such that the circuit shown below will behave exactly the same as the 2-port network from which the measurements were taken. Hint: use your answers above and the defining equations for 2-port networks.

$$R_1 = \quad \alpha = \quad R_2 = \quad \beta =$$
4. Fourier and Laplace Transforms (18 points)
Use the signals below to answer the following questions.

\[ x(t) = e^{-|t|} \cos(\omega t) \quad y(t) = 3 \cos(60\pi t) + 5 \sin(20\pi t) - 2\sqrt{2} \]

a. (2 pts) What is the Fourier transform of \( x(t) \)?

\[ X(\omega) = \]

b. (2 pts) What is the Laplace transform of \( x(t) \)?

\[ X(s) = \]

c. (3 pts) What is the Fourier transform of \( y(t) \)?

\[ Y(\omega) = \]

d. (1 pt) What is the inverse Fourier transform of \( Y(\omega) \) in part c.?

\[ y_{\text{inv}}(t) = \]

e. (3 pts) What is the Laplace transform of \( y(t) \)?

\[ Y(s) = \]

f. (1 pt) What is the inverse Laplace transform of \( Y(s) \) in part e.?

\[ y_{\text{inv}}(t) = \]

g. (1 pt) What is the fundamental frequency of \( y(t) \)? \( f_o = \) ______ Hz

h. (2 pts) What portion of the signal power of \( y(t) \) is in the range 1-25 Hz?

\[ P = \] ______% 

i. (3 pts) Consider the following Fourier series expansion that was used in lecture,

\[ y_{FS}(t) = a_o + \sum_{k=1}^{\infty} b_k \cos(k \omega_o t) + \sum_{k=1}^{\infty} c_k \sin(k \omega_o t) \]. What are the coefficients for \( y(t) \)?

\[ a_o = \quad b_k = \quad c_k = \]