Name or 6-Digit Code: __________________________
PSU Student ID Number: __________________________

Exam 2
February 27, 2006

ECE 222: Signals and Systems
Dr. McNames

• Write your full name above.
• Keep your exam flat during the entire exam.
• If you have to leave the exam temporarily, close the exam and leave it face down while you are out of the room.
• Turn off any cell phones or pagers that might interrupt the exam.
• Do not open the exam until instructed to do so.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages and write a note directing my attention to these pages.
• You will have 100 minutes to complete the exam.
• If you have extra time, double check your answers.
• Remember to include units with each of your answers.

Problem 1:______ /  14
Problem 2:______ /  12
Problem 3:______ /  17
Problem 4:______ /    7

Total:______ /  50
1. Fundamental Concepts (14 pts)

Circle the appropriate answers to the multiple choice questions below. Note that some questions may have more than one correct answer that must be circled. In these cases one point will be deducted for every incorrect answer, but the minimum score for any question is 0 pts.

a. (1 pt) What determines the initial condition of a capacitor (circle one)?
   - Voltage
   - Current
   - Both
   - Neither

b. (1 pt) What determines the initial condition of an inductor (circle one)?
   - Voltage
   - Current
   - Both
   - Neither

c. (1 pt) Suppose we take the Laplace transform of a signal \( X(s) = L\{x(t)\} \). What is the inverse transform of \( X(s) \)?
   \[ L^{-1}\{X(s)\} = \_________________________ \]

d. (2 pts) The defining equation for a capacitor in the time domain is \( i(t) = C \frac{dv(t)}{dt} \). What is the defining equation for a capacitor in the \( s \) domain?
   \[ I(s) = \_________________________ \]

e. (2 pts) List two properties that a circuit must satisfy in order to use transfer function analysis.

f. (1 pt) Consider two circuits with transfer functions of \( H(s) \) and \( G(s) \) where all the signals are voltages. Under what conditions is the transfer function of the cascade of the two circuits equal to \( H(s) \cdot G(s) \)?

\[ x(t) \rightarrow \underbrace{H(s)}_{\text{H}} \rightarrow y(t) \rightarrow \underbrace{G(s)}_{\text{G}} \rightarrow z(t) \quad x(t) \rightarrow \underbrace{H(s)}_{\text{H}} \rightarrow \underbrace{G(s)}_{\text{G}} \rightarrow z(t) \]

g. (1 pt) What is the relationship of the transfer function of a circuit and it’s impulse response?

h. (2 pts) What are the magnitude and fundamental period of the signal \( x(t) = -17 \exp(j5235t) \)?
   \[ |x(t)| = \_________ \quad T_0 = \_________ \]

i. (1 pt) What is the value of the following expression?
   \[ \int_{-\infty}^{\infty} \cos(0.25 \pi t) \delta(t - 16) \, dt = \_________________________ \]

j. (1 pt) Evaluate the following integral.
   \[ \int_{-\infty}^{\infty} \delta(t + 27) \, dt = \_________________________ \]

k. (1 pt) Write an expression for \( x(t) = 2\exp(0.3t+j0.2t) \) in rectangular form.
   \[ x(t) = \_________________________ \]
2. Laplace Transforms (12 points)

l. (4 pts) Find the Laplace transform of $x(t) = (t + 3)u(t - 3) + 3e^{-4t} + 4u(t + 5)$

$$X(s) =$$

m. (4 pts) Find the inverse Laplace transform of $X(s) = 12 - \frac{5}{s + 7} + \frac{12}{(s - 4)^2}$.

$$x(t) =$$

n. (4 pts) Find the inverse Laplace transform of $X(s) = \frac{5e^{-0.7\pi}}{s - 7 - j6} - \frac{5e^{-0.7\pi}}{s - 7 + j6}$.

$$x(t) =$$
3. Laplace Transform Circuit Analysis (17 pts)
Use the circuit shown below to answer the following questions.

![Circuit Diagram]

a. (1 pt) What is the magnitude of the initial voltage across the capacitor at $t = 0$?

$v_c(0) = \ldots$

b. (1 pt) What is the magnitude of the initial current through the inductor at $t = 0$?

$i_L(0) = \ldots$

c. (6 pts) Draw the $s$-domain equivalent circuit. Use the $s$-domain equivalent that includes a voltage source for both the capacitor and inductor, if appropriate. If you cannot solve for initial conditions, make up values and note these in your solution.

d. (9 pts) Use nodal analysis to write three independent equations in terms of the $s$-domain voltages $V_1$, $V_2$, and $V_3$. Do not include any other unknown variables in your equations. Do not simplify your equations.

Node 1:

Node 2:

Node 3:
4. Transfer Functions and Laplace Transform Circuit Analysis (7 pts)
Assume zero initial conditions.

a. (2 pts) Which elements in the circuit do not affect the transfer function \( H(s) = \frac{V_o(s)}{V_s(s)} \)?
   \( R_1 \quad R_2 \quad R_3 \quad C_1 \quad C_2 \quad C_3 \quad C_4 \)

b. (1 pt) If the output of this circuit was connected to another circuit, would it affect the transfer function \( H(s) = \frac{V_o(s)}{V_s(s)} \)? Explain.

c. (4 pts) Solve for the transfer function \( H(s) = \frac{V_o(s)}{V_s(s)} \). Hint: use nodal analysis. Simplify your expression as much as possible.

\[
H(s) = \]