Exam 2a Solutions
November 13, 2002

ECE 221: Electric Circuits
Dr. McNames

• Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
• Do not begin the exam or look at the problems until instructed to do so.
• You have 100 minutes to complete the exam.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages.
• If you have extra time, double check your answers. If you run out of time, write a note describing your strategy and equations that can be used to help solve the problem.

Problem 1:______ / 15
Problem 2:______ / 17
Problem 3:______ / 18

Total:______ / 50

First Letter in Last Name:______________

6-Digit Identification Number:______________

Student Identification Number:______________
1. **Thevenin Equivalents, Norton Equivalents, and Superposition (15 pts)**

Use the circuit below to answer the following questions. For partial credit, draw the appropriate circuit for each question.

![Circuit Diagram](image)

**a.** (2 pts) Use superposition to find the current that flows from terminal \( a \) to \( b \) when the terminals are connected (short-circuit) due to the voltage source acting alone.

\[
\begin{align*}
48 \text{ V} & \\
\begin{array}{c}
\downarrow \text{I}_{scv} \downarrow \\
4 \text{ k}\Omega & 4 \text{ k}\Omega \\
\downarrow & \downarrow \\
6 \text{ k}\Omega & 4 \text{ k}\Omega \\
\end{array}
\end{align*}
\]

Note: 6k resistor is in parallel with a short circuit. Thus, no current flows across this resistor. The short-circuit current is equal to the current flowing across the bottom 4 k resistor.

\[
I_{scv} = -\left( \frac{4k}{4k + 4k} \right) \frac{48}{4k + (4k || 4k)}
\]

\[I_{scv} = -4 \text{ mA}\]

**b.** (2 pts) Use superposition to find the current that flows from terminal \( a \) to \( b \) when the terminals are connected (short-circuit) due to the current source acting alone.

\[
\begin{align*}
& \\
& \begin{array}{c}
\downarrow \text{I}_{sci} \downarrow \\
4 \text{ k}\Omega & 4 \text{ k}\Omega \\
\downarrow & \downarrow \\
6 \text{ k}\Omega & 4 \text{ k}\Omega \\
\end{array}
\end{align*}
\]

Note: all of the 2 mA flows across the short circuit because this is the path of least resistance. You can also reach this conclusion by noting that there is no voltage drop across any of the resistors.

\[I_{sci} = 2 \text{ mA}\]

**c.** (1 pt) Find the Norton equivalent current.

\[I_N = I_{scv} + I_{sci} = -2 \text{ mA}\]
d. (1 pt) If the current source produced twice as much current as shown, what would the Norton equivalent current be? Hint: linearity.

\[ I_{N,2C} = I_{scv} + 2I_{sci} = 0 \text{ mA} \]

e. (1 pt) If the polarity of the voltage source was reversed, what would the Norton equivalent current be?

\[ I_{N,V} = -I_{scv} + I_{sci} = 6 \text{ mA} \]

f. (2 pts) Find the Thevenin/Norton equivalent resistance of the circuit.

\[ R_{eq} = 6k \parallel \left( (4k \parallel 4k) + 4k \right) = 3 \text{ k} \Omega \]
1. Thevenin Equivalents, Norton Equivalents, and Superposition Continued (15 pts)

The circuit on the previous page is repeated below for your convenience.

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\[
\begin{align*}
V_{Th} &= I_n \times R_{eq} = -6 \text{ V} \\
\end{align*}
\]

h. (2 pts) Draw both the Thevenin and Norton equivalents of the circuit as seen from the nodes \(a\) and \(b\). Clearly label these nodes.

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i. (1 pt) Suppose a resistor \(R_L\) is connected to the nodes \(a\) and \(b\). What value of \(R_L\) will maximize the power delivered to \(R_L\)?

\[R_L = R_{eq} = 3 \text{ k}\Omega\]

j. (1 pt) Suppose a resistor \(R_L\) is connected to the nodes \(a\) and \(b\). What is the maximum power that can be delivered to \(R_L\)?

\[
P_L = \frac{1}{4} I_n^2 R_{eq} = \frac{V_{Th}^2}{4R_{eq}} = 3 \text{ mW}
\]

k. (1 pt) Is this equal to half of the power produced by the voltage source and current source in the original circuit? (Circle)

Yes \[\Box\] No
2. Mesh Current Method (17 points)

a. (12 pts) Use the mesh-current method to write four independent equations in terms of the currents \(i_a, i_b, i_c,\) and \(i_d\). Do not use any other variables in your equations. If appropriate, use the supermesh technique. You do not need to simplify your equations.

Equation 1: \[700i_a + 500i_b + 2k(i_a - i_c) = 0\]

Equation 2: \[lm(2k(i_c - i_a)) = i_a - i_b\]

Equation 3: \[2k(i_c - i_a) + 1k \cdot i_d + 3k \cdot i_c = 0\]

Equation 4: \[4m = i_d - i_c\]

b. (4 pts) Solve for the currents \(i_a, i_b, i_c,\) and \(i_d\).

\[
\begin{bmatrix}
2.7 & 0.5 & -2 & 0 \\
-3 & 1 & 2 & 0 \\
-2 & 0 & 5 & 1 \\
0 & 0 & -1 & -1
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
i_d
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
4m
\end{bmatrix}
\]

\[i_a = -0.625 \text{ mA} \quad i_b = -0.125 \text{ mA} \quad i_c = -0.875 \text{ mA} \quad i_d = 3.13 \text{ mA}\]

c. (1 pt) How many independent equations would be necessary to solve for the node voltages using the node-voltage technique?

\[n = 3\] (There are 4 essential nodes)
3. Operational Amplifiers (18 points)

Use the circuit below to answer the following questions. The op amp is ideal.

![Circuit Diagram]

a. (1 pt) Does this op amp have negative feedback? (Circle)
   - Yes
   - No

b. (1 pt) Find $i_3$. 
   
   $i_3 = 0 \text{ mA} : \text{The op amp is ideal so no current can enter or leave the input terminals}$

c. (1 pt) Write an expression for the voltage at the inverting terminal of the op amp in terms of the appropriate resistances and $v_s$.
   
   $v_- = v_s \frac{R_2}{R_1 + R_2} : \text{Voltage divider}$

d. (1 pt) Write an expression for $v_o$ in terms of the appropriate resistances and $v_s$.
   
   $v_o = v_s \left( \frac{R_2}{R_1 + R_2} \right) = v_- = v_+$

e. (1 pt) Which resistors affect the relationship of $v_o$ and $v_s$?
   - $R_1$
   - $R_2$
   - $R_3$
   - $R_4$

f. (1 pt) If we used the real model of an op amp, which resistors would affect the relationship of $v_o$ and $v_s$?
   - $R_1$
   - $R_2$
   - $R_3$
   - $R_4$

g. (1 pt) If we used the real model of an op amp and $R_4 = 0$, what would $v_o$ be?
   
   $v_o = 0 \text{ V} : \text{The output would be connected to a short circuit in this case.}$
3. **Operational Amplifiers Continued (18 points)**

Use the circuit below to answer the following questions. The op amps in the circuit are ideal. The questions are intended to be answered in the order listed. For partial credit, include algebraic expressions for each answer next to the corresponding question. Each expression may include variables listed in earlier questions.

![Circuit Diagram](image)

- **h. (1 pt) Find \( v_b \).**
  \[ v_b = 7 \text{ V} \]

- **i. (1 pt) Find \( v_d \).**
  \[ v_d = 15 \text{ V} \]

- **j. (1 pt) Find \( i_c \).**
  \[ i_c = \frac{v_d - v_b}{9k} = 0.889 \text{ mA} \]

- **k. (1 pt) Find \( i_d \).**
  \[ i_d = \frac{v_d}{4k} = 3.75 \text{ mA} \]

- **l. (1 pt) Find \( i_b \).**
  \[ i_b = i_d + i_c = 4.64 \text{ mA} \]

- **m. (2 pts) Find \( v_c \).**
  \[ v_c = i_b \cdot 2k + v_d = 24.3 \text{ V} \]

- **n. (1 pt) Find \( i_e \).**
  \[ i_e = \frac{v_c - v_b}{7k} = 2.47 \text{ mA} \]

- **o. (1 pt) Find \( i_a \).**
  \[ i_a = i_e + i_c = 3.36 \text{ mA} \]

- **p. (2 pts) Find \( v_a \).**
  \[ v_a = -i_a \cdot 5k + v_b = -9.79 \text{ V} \]