Exam 1b Solutions
October 23, 2002

ECE 221: Electric Circuits
Dr. McNames

• Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
• Do not begin the exam or look at the problems until instructed to do so.
• You have 100 minutes to complete the exam.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages.
• If you have extra time, double check your answers. If you run out of time, write a note describing your strategy and equations that can be used to help solve the problem.

Problem 1:______ / 13
Problem 2:______ / 8
Problem 3:______ / 15
Problem 4:______ / 14

Total:______ / 50

First Letter in Last Name:________________

6-Digit Identification Number:______________

Student Identification Number:______________
1. Fundamental Concepts (13 pts)  
Use the following table and circuit diagram to answer the following questions. The column labeled PSC represents whether the passive sign convention (PSC) is satisfied for each circuit element.

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (I)</th>
<th>PSC (Y/N)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-8</td>
<td>3</td>
<td>N</td>
</tr>
<tr>
<td>b</td>
<td>7</td>
<td>0</td>
<td>Y</td>
</tr>
<tr>
<td>c</td>
<td>-12</td>
<td>3</td>
<td>N</td>
</tr>
<tr>
<td>d</td>
<td>20</td>
<td>-5</td>
<td>Y</td>
</tr>
<tr>
<td>e</td>
<td>-5</td>
<td>2</td>
<td>N</td>
</tr>
<tr>
<td>f</td>
<td>15</td>
<td>-2</td>
<td>Y</td>
</tr>
<tr>
<td>g</td>
<td>-10</td>
<td>-4</td>
<td>Y</td>
</tr>
<tr>
<td>h</td>
<td>5</td>
<td>-4</td>
<td>N</td>
</tr>
</tbody>
</table>

![Circuit Diagram]

a. (8 pts) Fill in the missing values in the table above.

b. (1 pt) Which circuit elements could be voltage sources? (Circle)
   ![Circuit Diagram with Circles]

c. (1 pt) Which circuit elements could be resistors? Recall that resistors can only dissipate power. (Circle)
   ![Circuit Diagram with Circles]

d. (1 pt) What is the total power absorbed in the circuit.
   \[ P_{\text{abs}} = 130 \text{ W} \]

e. (1 pt) How many meshes are in the circuit?
   \[ m = 4 \]

f. (1 pt) How many essential nodes are in the circuit?
   \[ n = 4 \]
2.  **Resistive Networks (8 pts)**

Find the equivalent resistance of the circuits shown below.

a.  (2 pts)

\[ R_{eq} = 5k + \left( \frac{12k}{20k} \parallel \left( \frac{2k}{(9k \parallel 7k)} \right) \right) \]

\[ R_{eq} = 8.31 \, \text{k}\Omega \]

b.  (2 pts)

\[ R_{eq} = 2.5k + \left( \frac{9k}{(5k \parallel 4k)} \right) + 3k \]

\[ R_{eq} = 10.0 \, \text{k}\Omega \]
2. Resistive Networks Continued (8 pts)
Find the equivalent resistance of the circuits shown below.

c. (2 pts)

\[
R_{eq} = 2k + \left(8k || (3k || 10k) + (3k || 5k) + 5k\right) + 1k
\]

\[R_{eq} = 7.28 \text{k}\Omega\]

d. (2 pts)

\[
R_{eq} = 3k + \left((3k + 3k) || (3k + 5k + 1k)\right)
\]

\[R_{eq} = 6.60 \text{k}\Omega\]
3. Mesh Current Method (15 points)

a. (11 pts) Use the mesh-current method to write three independent equations in terms of the currents $i_a$, $i_b$, and $i_c$. Do not use any other variables in your equations. If appropriate, use a supermesh. You do not need to simplify your equations.

Equation 1: $1000(i_c - i_a) + 5k \cdot i_a + 7k (i_a - i_c) + 4k(i_a - i_b) = 0$

Equation 2: $3k \cdot i_b + 4k (i_b - i_a) + 7k(i_c - i_a) + 5k \cdot i_c = 0$

Equation 3: $i_c - i_b = 3m$

b. (3 pts) Solve for the currents $i_a$, $i_b$, and $i_c$.

\[
\begin{bmatrix}
15 & -4 & -6 \\
-11 & 7 & 12 \\
0 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
3m
\end{bmatrix}
\]

\[
i_a = -0.103 \text{ mA} \quad i_b = -1.95 \text{ mA} \quad i_c = 1.05 \text{ mA}
\]

c. (1 pt) How much power is being absorbed by the dependent source?

\[
P_{1000} = i_a \cdot 1000(i_c - i_a) = -118 \text{ µW}
\]
4. Node Voltage Method & Supernodes (14 points)

a. (9 pts) Use the node-voltage method to write three independent equations in terms of the node voltages $v_1$, $v_2$, and $v_3$. Do not use any other variables in your equations. If appropriate, use a supernode. You do not need to simplify your equations.

Eq. 1: 
$$-3 \left( \frac{v_2 - v_3}{9} \right) + \frac{v_1}{60} + \frac{v_1 - v_2}{18} = 0$$

Eq. 2: 
$$\frac{v_2 - v_1}{18} + \frac{v_2 - v_3}{80} + \frac{v_2 - v_3}{9} = 0$$

Eq. 3: 
$$\frac{v_3 - v_2}{9} + \frac{v_3 - 3(v_1 - v_2)}{8} + \frac{v_3 - 30}{12} = 0$$

b. (3 pts) Solve for the node voltages $v_1$, $v_2$, and $v_3$.

$$\begin{bmatrix} \frac{1}{60} + \frac{1}{18} & -\frac{3}{9} & \frac{2}{9} \\ -\frac{1}{18} & \frac{1}{18} + \frac{1}{80} & -\frac{1}{9} \\ -\frac{1}{8} & -\frac{3}{8} & \frac{1}{9} + \frac{4}{15} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{30}{12} \end{bmatrix}$$

$$v_1 = -63.0 \text{ V} \quad v_2 = -40.1 \text{ V} \quad v_3 = -33.1 \text{ V}$$

c. (1 pt) What is $i_a$?

$$i_a = \frac{v_2 - v_3}{9} = -776 \text{ mA}$$

d. (1 pt) What is $v_\beta$?

$$v_\beta = v_1 - v_2 = -23.0 \text{ V}$$