Final Exam
March 14, 2000

ECE 221: Electric Circuits
Dr. McNames

- Write your 6-digit identification number and student identification numbers below.
- Do not begin the exam or look at the problems until instructed to do so.
- You have 100 minutes to complete the exam.
- Once you begin, write your student ID at the top of each page and make sure you have all the pages.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages.
- Do not expect your answers to be tidy values like the homework problems.
- If you have extra time, double check your answers. If you run out of time, write the relevant equations that can be used to help solve the problem and a note describing your approach.

Problem 1:______ / 20
Problem 2:______ / 15
Problem 3:______ / 15

Total:______ / 50

6-Digit Identification Number:_____________

Student Identification Number:_____________
1. Thevenin, Norton, & Maximum Power Transfer (20 Points)

a. (3 pts) For the circuit shown above, write the node-voltage equation at node 1 in terms of $V_1$ and $V_a$ when the load is replaced with an open circuit. You do not need to simplify your equation.

Eqtn: 
\[
\begin{align*}
V_1 - \frac{250}{50 - j100} + 0.01V_a + \frac{V_1 - V_a}{100} &= 0
\end{align*}
\]

b. (2 pts) Write an expression for $V_a$ in terms of $V_1$ that is independent of the equation written above when the load is replaced with an open circuit.

\[
V_a = \frac{j50}{100 + j50} V_1
\]

c. (2 pts) Solve the two equations above for the values of $V_1$ and $V_a$.

\[
\begin{align*}
V_1 &= 138.7 \angle 33.69^\circ \text{ V} \\
V_a &= 62.02 \angle 97.13^\circ \text{ V}
\end{align*}
\]

d. (1 pt) What is the open circuit voltage between the terminals $a$ and $b$?

\[
V_{oc} = 62.02 \angle 97.13^\circ \text{ V}
\]

e. (2 pts) Solve for the current that flows from terminal $a$ to $b$ when the load is replaced with a direct connection (short circuit current).

\[
I_{sc} = \frac{250}{50 - j100 + 100}
\]

\[
I_{sc} = 1.387 \angle 33.69^\circ \text{ V}
\]
1. **Continued (20 Points)**
The circuit from the previous page is repeated below for convenience.

![Circuit Diagram]

f. (1 pt) What is the Thevenin equivalent impedance seen at the terminals $a$ and $b$? Hint: you may use your answers to the previous questions.

$$Z_{Th} = \frac{V_{OC}}{I_{SC}} = 20 + j40 \, \Omega$$

g. (4 pts) Draw the Thevenin and Norton equivalent circuits seen at the terminals $a$ and $b$.

![Equivalent Circuits]

h. (1 pt) What is the load impedance that will maximize the power dissipated in the load?

$$Z_L = Z_{Th}^* = 20 - j40 \, \Omega$$

i. (1 pt) How much power is dissipated in the load when $Z_L$ has the value specified in part h.?

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = 48.08 \, \text{W}$$

j. (2 pts) What are the values of the voltage and current across the load when it has the value specified in part h.?

$$V_{ab} = 69.34\angle33.69^\circ \, \text{V} \quad I_{ab} = 1.55\angle97.13^\circ \, \text{A}$$

k. (1 pt) What is the reactive power dissipated in the load when $Z_L$ has the value specified in part h.?

$$Q = \text{Im}\{V_{ab} I_{ab}^*\} = -96.15 \, \text{VAR}$$
2. **Mesh Current Method (15 pts)**

![Circuit Diagram]

a. (6 pts) For the circuit shown above, write the mesh-current equations around the two loops shown in terms of the currents $I_a$ and $I_b$. You do not need to simplify your equations.

Loop $I_a$: $j8I_a + 16(I_a + 2) - j4(I_a - I_b) = 0$

Loop $I_b$: $-j4(I_b - I_a) + j12(I_b + 2) = 10\angle45^\circ$

b. (2 pts) Solve for the currents $I_a$ and $I_b$.

$I_a = 2.405\angle149.3^\circ$ A \hspace{1cm} $I_b = 3.220\angle-152.3^\circ$ A

c. (1 pt) How much complex power is being delivered by the independent voltage source?

$S_{10V} = (10\angle-45)(I_b^*) = -9.558 + j30.75$ VA

d. (2 pts) How much power is being delivered by the independent current source?

$S_{2A} = (-j8 I_a + 10\angle-45)(2\angle0^\circ) = 33.81 + j18.93$ VA

e. (2 pts) How much power is being absorbed by the resistor?

$P_R = \text{Re}\{S_{10V}\} + \text{Re}\{S_{2A}\} = 24.25$ W

f. (2 pts) How much total reactive power is being absorbed by the capacitor and both inductors?

$Q_{C2L} = \text{Im}\{S_{10V}\} + \text{Im}\{S_{2A}\} = 49.69$ VAR
3. Linear Transformers (15 points)

a. (1 pt) What is the self impedance of the primary circuit?
\[ Z_{11} = 1k + j200 + 400 + j7.2k = 1.4k + j7.4k \Omega \]

b. (1 pt) What is the self impedance of the secondary circuit?
\[ Z_{22} = 200 + 1.6k + j3.2k - j5k = 1.8k - j1.8k \Omega \]

c. (2 pts) What is the impedance reflected into the primary winding?
\[ Z_{R1} = \frac{(\omega M)^2 |Z_{22}|}{|Z_{11}|^2} = 1.6k + j1.6k \Omega \]

d. (1 pt) What is the impedance seen looking into the terminals \(a\) and \(b\) of the transformer?
\[ Z_{ab} = Z_{R1} + 400 + j7.2k = 2k + j8.8k \Omega \]

e. (2 pts) What is the impedance seen at the terminals \(c\) and \(d\) when the independent voltage source is replaced with a short circuit?  Hint: this is the Thevenin impedance seen by the load.
\[ Z_{R2} = \frac{(\omega M)^2 |Z_{11}|}{|Z_{R1}|^2} = 142.2 - j751.5 \Omega \]
\[ Z_{Th} = Z_{R2} + 200 + j3.2k = 342.2 + j2449 \Omega \]

f. (2 pts) What is the voltage \(V_2\) when \(I_2 = 0\)?  Hint: this is the open circuit, or Thevenin, voltage at the terminals \(c\) and \(d\).
\[ I_1 = \frac{250 \angle 0^\circ}{1.4k + j7.4k}, \quad V_2 = (j2.4k) \cdot I_1 = V_{oc} = V_{Th} \]
\[ V_{Th} = 79.67 \angle 10.71^\circ V \]

g. (2 pts) What is the current \(I_1\) for the load shown?
\[ I_1 = \frac{250 \angle 0^\circ}{Z_{11} + Z_{R1}} = 26.35 \angle -71.51^\circ mA \]

h. (2 pts) What is the current \(I_2\) for the load shown?
\[ I_2 = \frac{V_{Th}}{Z_{Th} + (1.6k - j5k)} = 4.85 \angle 63.43^\circ mA \]

i. (2 pts) What is the voltage at the secondary terminals \(c\) and \(d\) for the load shown?
\[ V_2 = I_2 (1.6k - j5k) = 130.4 \angle -8.82^\circ V \]