Exam 1 Solutions
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ECE 221: Electric Circuits
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• Write your 6-digit identification number and student identification numbers below.
• Do not begin the exam or look at the problems until instructed to do so.
• You have 100 minutes to complete the exam.
• Once you begin, write your student ID at the top of each page.
• Do not use separate scratch paper. If you need more space, use the backs of the exam pages.
• If you have extra time, double check your answers. If you run out of time, write a note describing your strategy and equations that can be used to help solve the problem.

Problem 1:______ / 17
Problem 2:______ / 20
Problem 3:______ / 13
Problem 4:______ / 10

Total:______ / 60

6-Digit Identification Number:_________________

Student Identification Number:_________________
1. Mesh Current Method (17 points)

![Circuit Diagram]

a. (9 pts) For the circuit shown above, write the mesh-current equations around each loop in terms of the currents $i_a$, $i_b$, and $i_c$. You do not need to simplify your equations.

Loop $i_a$: $10i_a + 2 \cdot 40i_c + 50(i_a - i_c) = 0$

Loop $i_b$: $-2 \cdot 40i_c + 60i_b + 80(i_b - i_c) = 0$

Loop $i_c$: $50(i_c - i_a) + 80(i_c - i_b) + 40i_c = 20$

b. (3 pts) Solve for the currents $i_a$, $i_b$, and $i_c$.

$i_a = -96.6 \text{ mA} \quad i_b = 221 \text{ mA} \quad i_c = 193 \text{ mA}$

c. (2 pts) How much power is being delivered by the independent source?

$P_{20V} = 20i_c = 3.86 \text{ W}$

d. (2 pts) How much power is being delivered by the dependent source?

$P_{2V1} = 2 \cdot v_1(i_b - i_a) = 2 \cdot 40i_c(i_b - i_a) = 4.90 \text{ W}$

e. (1 pt) How much power is being absorbed by the resistors?

$P_R = P_{20V} + P_{2V1} = 8.76 \text{ W}$
2. Node Voltage Method (20 pts)

a. (6 pts) For the circuit shown above, write the node-voltage equations at each of the specified nodes in terms of $v_1$, $v_3$, $v_4$, and $i_{14}$. You do not need to simplify your equations.

Node 1: \[ \frac{v_1}{50} + \frac{v_1 - 20}{4} + i_{14} = 0 \]

Node 3: \[ \frac{v_3 - 20}{6} + \frac{v_3}{40} + \frac{v_3 - v_4}{2} = 0 \]

Node 4: \[ \frac{v_4 - v_3}{2} + \frac{v_4}{20} - i_{14} = 0 \]

b. (1 pt) Write an expression for $i_\alpha$ in terms of $v_3$ and $v_4$.

\[ i_\alpha = \frac{v_4 - v_3}{2} \]

c. (2 pts) In part a, you should have written 3 independent equations in 4 unknowns. Write a fourth independent equation that relates the voltages between nodes 1 and 4 to the voltages $v_3$ and $v_4$. Hint: use your answer from part b.

\[ \text{Eq: } \frac{30(v_4 - v_3)}{2} = v_4 - v_1 \]
2. Node Voltage Method – Continued

The same circuit from the previous page is shown below for convenience.

![Circuit Diagram](image)

**d. (6 pts)** Use a supernode to write three independent equations in terms of $v_1$, $v_3$, and $v_4$. You do not need to simplify your equations.

\[
\text{Eq. 1: } \frac{v_1}{50} + \frac{v_1 - 20}{4} + \frac{v_4 - v_3}{2} + \frac{v_4}{20} = 0
\]

\[
\text{Eq. 2: } \frac{v_3 - 20}{6} + \frac{v_3}{40} + \frac{v_3 - v_4}{2} = 0
\]

\[
\text{Eq. 3: } \frac{30(v_4 - v_1)}{2} = v_4 - v_1
\]

e. **(3 pts)** Solve for the voltages $v_1$, $v_3$, and $v_4$.

$v_1 = 14.7 \text{ V}$  
$v_3 = 18.0 \text{ V}$  
$v_4 = 18.2 \text{ V}$

**f. (2 pts)** How much power is being delivered by the dependent source?

\[
i_a = \frac{v_4 - v_3}{2} = 115 \text{ mA}
\]

\[
P_{30} = 30i_a \cdot i_a = 30i_a \left( i_a + \frac{v_4}{20} \right) = 3.54 \text{ W}
\]
3. Thevenin/Norton Equivalents & Maximum Power Transfer (13 pts)

Hint: think carefully about how to approach this problem. Some approaches are easier than others.

![Circuit Diagram](image)

a. (4 pts) Find the current flowing from the node labeled $a$ to the node labeled $b$ if the nodes are connected together (short circuit current).

Solve part b. first, then calculate $I_{sc}$ by the relationship of $R_{eq}$ to $V_{oc}$.

$$R_{eq} = (20k || 30k) + 8k = 20k\Omega$$

$$I_{sc} = \frac{V_{oc}}{R_{eq}} = -1.2 \text{ mA}$$

b. (4 pts) Find the voltage between the terminals $a$ and $b$ if the nodes are left unconnected (open circuit voltage), as shown. Label the voltage across the 30k Ohm resistor as $V$ and apply the node-voltage method.

$$\frac{V - 40}{20k} + \frac{V}{30k} - 2m = 0; \quad V = 48 \text{ V}$$

$$V_{oc}=40 - V - 2m \cdot 8k = -24 \text{ V}$$

c. (4 pts) Draw the Thevenin and Norton equivalents of the circuit as seen from the nodes $a$ and $b$. Clearly label these nodes.

![Thevenin and Norton Equivalents](image)

d. (1 pt) Suppose a resistor $R_L$ is connected to the nodes $a$ and $b$. What value of $R_L$ will maximize the power delivered to $R_L$?

$$R_L = 20 \text{ k}\Omega$$
4. Operational Amplifiers (10 pts)
For both circuits below, assume that the operational amplifier is ideal and operating in the linear region.

a. (4 pts) Find the values of $R_f$ and $R_b$ so that $v_o = 20i_b - 10i_a$.

\[
v_p = i_b R_b
\]
\[
\frac{v_o - v_p}{R_f} = -i_a
\]
\[
v_o = R_b i_b - R_f i_a
\]

$R_b = 20 \, \Omega$ \hspace{1cm} $R_f = 10 \, \Omega$

b. (6 pts) Find an expression for $v_o$ as a function of $v_f$ and $i_2$.

\[
v_f = -R_4 i_2
\]
\[
\frac{v_f}{R_3} = \frac{v_o - v_1}{R_3 + R_1} = -\frac{R_4 i_2}{R_3}
\]
\[
v_o = v_1 - \left(\frac{R_3 + R_1}{R_3}\right) R_4 i_2
\]