Exam 2 Solutions
November 15, 2000

- Write the first letter in your last name, your 6-digit identification number, and your student identification number below.
- Do not begin the exam or look at the problems until instructed to do so.
- You have 100 minutes to complete the exam.
- Once you begin, write your student ID at the top of each page.
- Do not use separate scratch paper. If you need more space, use the backs of the exam pages.
- If you have extra time, double check your answers. If you run out of time, write a note describing your strategy and equations that can be used to help solve the problem.

Problem 1:______ / 10
Problem 2:______ / 16
Problem 3:______ / 18
Problem 4:______ / 26

Total:______ / 70

First Letter in Last Name:_______________

6-Digit Identification Number:_____________

Student Identification Number:_____________
1. **Fundamental Concepts (10 pts)**

Circle either True or False for each of the questions below.

- True  False  Inductors in parallel can be combined just like resistors in parallel.

- True  False  The energy stored in an inductor can change instantly.

- True  False  The energy stored in a capacitor can change instantly.

- True  False  Resistors store energy in the form of an electric field.

- True  False  Capacitors store energy in the form of a magnetic field.

- True  False  The ideal model of an operational amplifier is easier to analyze than the real model.

- True  False  The ideal model of an operational amplifier is more accurate than the real model.

- True  False  Phasors are useful for finding the transient sinusoidal response of a circuit.

- True  False  Operational amplifiers were given their name because they can be used to perform mathematical operations.

- True  False  The open loop voltage gain of operational amplifiers is typically small enough that it can be approximated as zero.
2. Mesh Current Analysis & Supermeshes (16 points)

a. (1 pt) What is the value of the mesh current $i_d$?

$$i_d = 3 \text{ A}$$

b. (9 pts) Use a supermesh to write three independent equations in terms of $i_a$, $i_b$, and $i_c$. Do not use the variables $i$ or $i_d$. You do not need to simplify your equations.

Equation 1: Supermesh: 
$$5i_a + 6i_b + 2(i_b - i_c) + 7(i_a - 3) = 0$$

Equation 2: Mesh $i_c$: 
$$4i_c - 5 + 3(i_c - 3) + 2(i_c - i_b) = 0$$

Equation 3: Dependent Source: 
$$i_b - i_a = 1.5i_c$$

c. (3 pts) Solve for the currents $i_a$, $i_b$, and $i_c$.

$$i_a = -0.100 \text{ A} \quad i_b = 3.35 \text{ A} \quad i_c = 2.30 \text{ A}$$

d. (1 pt) How much total power is being delivered by both independent sources?

$$P_i = P_{3A} + P_{5V} = 3(V_{3A}) + 5(V_{5V}) = 3(5i_a + 6i_b + 4i_c - 5) + 5(i_c) = 82.90 \text{ W}$$

e. (1 pt) How much power is being delivered by the dependent source?

$$P_d = I_d(V_d) = 1.5i_c(6i_b + 2(i_b - i_c)) = 76.59 \text{ W}$$

f. (1 pt) How much power is being absorbed by the resistors?

$$P_R = P_i + P_d = 159.5 \text{ W}$$
3. Operational Amplifiers (18 pts)

For the circuit below, assume that the operational amplifier is ideal and operating in the linear region.

![Circuit Diagram]

a. (1 pt) What is the value of \( v_n \)?

\[ v_n = 0 \text{ V} \]

b. (1 pt) What is the value of \( i_2 \)?

\[ i_2 = \frac{v_n}{R_2} = 0 \text{ A} \]

c. (2 pts) Write an expression that relates \( v_1 \) to \( v_o \).

\[ \frac{v_1}{R_1} + \frac{v_o}{R_3} = 0 \]

\[ v_1 = -\frac{R_1}{R_3} v_o \]

d. (3 pts) Write the node voltage equation at the node labeled \( v_1 \).

\[ -I_s + \frac{v_1}{R_1} + \frac{v_1 - v_o}{R_4} = 0 \]

e. (2 pts) Solve for \( v_o \) in terms of \( I_s \). Simplify your expression as much as possible.

\[ I_s = -\frac{R_1}{R_4} v_o - \frac{R_1}{R_4} v_o - v_o \left( -\frac{R_4}{R_3 R_4} - \frac{R_1}{R_3 R_4} - \frac{R_1}{R_3 R_4} \right) \]

\[ I_s = v_o \left( -\frac{R_4 + R_1 + R_3}{R_3 R_4} \right) \]

\[ v_o = -\left( \frac{R_3 R_4}{R_4 + R_1 + R_3} \right) I_s \]
3. Operational Amplifiers Continued (18 pts)

For the circuit below, assume that the operational amplifier is ideal and operating in the linear region.

![Circuit Diagram]

f. (9 pts) Fill in the table below.

<table>
<thead>
<tr>
<th>Currents</th>
<th>Voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1 = 8$ mA</td>
<td>$v_1 = -4$ V</td>
</tr>
<tr>
<td>$i_2 = 8$ mA</td>
<td>$v_2 = 2.5$ V</td>
</tr>
<tr>
<td>$i_3 = -4$ mA</td>
<td>$v_3 = 2.5$ V</td>
</tr>
<tr>
<td>$i_4 = 0$ A</td>
<td></td>
</tr>
<tr>
<td>$i_5 = 1$ mA</td>
<td></td>
</tr>
<tr>
<td>$i_6 = -13$ mA</td>
<td></td>
</tr>
</tbody>
</table>

- $i_1 = 10 mA \left( \frac{8k}{8k + 2k} \right)$ Current Divider
- $i_2 = i_1$ Infinite Input Resistance & KCL
- $v_1 = -500 \times i_2$ Ohm’s Law & Negative Feedback
- $i_3 = v_1/(1 \, k)$ Ohm’s Law
- $i_4 = 0$ Infinite Input Resistance
- $v_2 = 5 \times 5 \, k/(5k + 5k)$ Voltage Divider
- $v_3 = v_2$ Voltage Follower (Negative Feedback)
- $i_5 = v_3/(2.5 \, k)$ Ohm’s Law
- $i_6 = \frac{v_1 - v_3}{500}$ Ohm’s Law
4. **RL and RC Circuits (26 pts)**

Use the circuit below to answer the following questions. The switches have been in their initial positions for a long time. Use the following page as a workspace.

![Circuit Diagram]

a. (18 pts) Fill in the table below. $w_L$ represents the energy stored in the inductor and $w_c$ represents the energy stored in the capacitor.

<table>
<thead>
<tr>
<th></th>
<th>Immediately before $t = 0$.</th>
<th>Immediately after $t = 0$.</th>
<th>Long after $t = 0$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t = 0-$</td>
<td>$t = 0+$</td>
<td>$t \rightarrow \infty$</td>
</tr>
<tr>
<td>$i_L$ = 10 mA</td>
<td>$i_L$ = 10 mA</td>
<td>$i_L$ = 1.25 mA</td>
<td></td>
</tr>
<tr>
<td>$v_L$ = 0 V</td>
<td>$v_L$ = -70 V</td>
<td>$v_L$ = 0 V</td>
<td></td>
</tr>
<tr>
<td>$w_L$ = 200 µJ</td>
<td>$w_L$ = 200 µJ</td>
<td>$w_L$ = 3.125 µJ</td>
<td></td>
</tr>
<tr>
<td>$i_c$ = 0 A</td>
<td>$i_c$ = -18.33 mA</td>
<td>$i_c$ = 0 A</td>
<td></td>
</tr>
<tr>
<td>$v_c$ = 60 V</td>
<td>$v_c$ = 60 V</td>
<td>$v_c$ = 5 V</td>
<td></td>
</tr>
<tr>
<td>$w_c$ = 45 mJ</td>
<td>$w_c$ = 45 mJ</td>
<td>$w_c$ = 312.5 µJ</td>
<td></td>
</tr>
</tbody>
</table>

b. (1 pt) What is the time constant for the portion of the circuit that contains the inductor?
   \[
   \tau_L = \frac{L}{R} = \frac{4}{8k\Omega} = 0.5 \text{ ms}
   \]

c. (1 pt) What is the time constant for the portion of the circuit that contains the capacitor?
   \[
   \tau_c = RC = \left(6k \parallel 6k\right)\frac{1}{C} = 75 \text{ ms}
   \]

d. (6 pts) Write the expressions for the voltages across and the currents in the inductor and capacitor for $t \geq 0$.
   All of these can be solved by applying the general equation: $x(t) = x_f + (x_i - x_f) e^{-\frac{t}{\tau}}$.

   \[
   i_L(t) = 1.25 + (10 - 1.25) e^{-2000t} \text{ mA}
   \]
   \[
   v_L(t) = -70 e^{-2000t} \text{ V}
   \]
   \[
   i_c(t) = -18.33 e^{-13.33t} \text{ mA}
   \]
   \[
   v_c(t) = 5 + (60 - 5) e^{-13.33t} \text{ V}
   \]
4. RL and RC Circuits Workspace (26 pts)

Use this page as a workspace for problem 4. The circuit is repeated below for convenience.

For $t = 0-$, the inductor acts like a short circuit and the capacitor acts like an open circuit. Thus $v_L = 0$ and $i_c = 0$. $i_L$ can be found by a current divider equation ($10 \, \text{k} \parallel 10 \, \text{k}$) and $v_c$ can be found by Ohm’s law.

Since $i_L$ and $v_c$ cannot change instantly, at $t = 0+$ the inductor acts like an ideal current source and the capacitor acts like an ideal voltage source. The circuit shown above can be used to solve for $v_L$ and $i_c$. Notice 10 V ideal source effectively isolates the left half of the circuit containing the inductor from the right half containing the capacitor. It may be easier to visualize this if the two halves are drawn separately, as shown below as $t \to \infty$.

For $t \to \infty$, the inductor acts like a short circuit again and the capacitor acts like an open circuit again. Thus $v_L = 0$ and $i_c = 0$, as for $t = 0-$. $i_L$ can be found by Ohm’s law ($10/8 \, \text{k}$) and $v_c$ can be found by the voltage divider equation $10 \times 6\, \text{k}/(2\, \text{k} + 4\, \text{k} + 6\, \text{k})$.

The figure above can also be used to determine the resistance seen by the capacitor and inductor. To do this, you must set the 10 V source to zero (short circuit).