Ideal Voltage Sources: Parallel

Technically allowed if $V_1 = V_2$, but is a bad idea.

- Ideal sources do not exist.
- Could easily cause component failure (smoke).
- In practice, the stronger source would win.
- Immovable object meets unstoppable force.
- How to measure the specified potential (voltage).
- Recall: Ideal voltage sources guarantee the voltage between two terminals is at the specified potential.
- Ideal sources do not exist.
  - Technically allowed if $V_1 = V_2$, but is a bad idea.
  - Ideal sources do not exist.
  - Could easily cause component failure (smoke).
  - In practice, the stronger source would win.
  - Immovable object meets unstoppable force.
  - How to measure the specified potential (voltage).

Smokes

Ideal Voltage Sources: Series

- Ideal voltage sources connected in series add.

$V_1 + V_2 = V_{total}$

Ideal Current Sources: Series

- Ideal current sources cannot be connected in series.

Ideal Current Sources: Parallel

- Ideal current sources cannot be connected in parallel.

$I_1 = I_2 = I_{total}$
Resistance: Basic Concepts & Assumptions

- We will always measure resistance in Ohms.
- Ohms are denoted by the greek letter Omega: \( \Omega \)
- Examples: 50 \( \Omega \), 1k \( \Omega \), 2.5M \( \Omega \).

- Conductors (e.g. wires) have very low resistance (< 0.1 Ohm) that can usually be ignored (i.e. we will assume wires have zero resistance).
- Insulators (e.g. air) have very large resistance (> 50M Ohm) that can usually be ignored (omitted from circuit for analysis).
- Resistors have a medium range of resistance and must be accounted for in the circuit analysis.
- Conceptually, a light bulb is similar to a resistor.

- Properties of the bulb control how much current flows and how much power is dissipated (absorbed & emitted as light and heat).
- Ohm’s law: \( \frac{V}{I} = \frac{V}{I} \) (where \( V \) is voltage, \( I \) is current).

Ohm’s Law

- As with all circuit elements, we need to know how the current through and voltage across the device are related.
- Many materials have a complicated nonlinear relationship between voltage and current.
- Ideal current sources in parallel add: \( I_1 + I_2 = I_1 + I_2 \).

Ideal Current Sources: Parallel

- Linear (Ohm’s Law Applies)
- Nonlinear (Ohm’s Law Does Not Apply)

- Materials with a linear current-voltage relationship satisfy Ohm’s law: \( I = \frac{V}{R} \).
- Many materials have a complicated nonlinear relationship:
  \( V = f(I) \).

- Materials with a linear relationship satisfy Ohm’s law: \( V = IR \).
- The slope, \( m \), is equal to the resistance of the element: \( V = mI \).
- As with all circuit elements, we need to know how the current and voltage are related.

Resistance: Defined

- All materials resist the flow of current.
- The current through and voltage across the device are related.
- Resistance is usually represented by the symbol \( R \).
- Resistance is a property of the material, not the circuit.
- Resistance is defined as the ratio of voltage to current:
  \[ R = \frac{V}{I} \]
- where
  \[ \frac{V}{I} = \frac{V}{I} \] (Ohms Law).
- Example: 50 \( \Omega \), 1k \( \Omega \), 2.5M \( \Omega \).
- Ohms are denoted by the Greek letter Omega: \( \Omega \).
- We will always measure resistance in Ohms.

Resistance: Defined

- A conductor of a given length and cross-sectional area has a resistance:
  \[ R = \frac{\rho L}{A} \]
- where
  \[ \frac{V}{I} = \frac{V}{I} \] (Ohm’s Law).
- \( \rho \) is the resistivity of the material in ohm-meters.
- \( L \) is the length of the material in meters.
- \( A \) is the cross-sectional area of the material in square meters.

- Current is the flow of charge per unit time.
- A current source is a device that supplies a constant current.
- Ideal current sources in parallel add:
  \[ I_1 + I_2 = I_1 + I_2 \].
Resistors & Passive Sign Convention

- **Clutter**
  - Could draw a resistor with \( R = 0 \), but is unnecessary and adds clutter
  - Often just drawn as a wire (line)

  An element (or wire) with \( R = 0 \) is called a short circuit

\[
\begin{array}{c}
\text{Circuit with } \! R = 0 \! \\
\text{current is unaltered}
\end{array}
\]

**Example 1: Ohm’s Law**

\[
\begin{align*}
\text{v} & = 0.5882 \text{ V} \\
\text{R} & = 10 \Omega \\
\text{i} & = 5 \text{ mA} \\
\text{v} & = 10.59 \text{ V} \\
\text{v} & = 13.53 \text{ V} \\
\text{i} & = 1.61 \text{ mA} \\
\text{i} & = 3.38 \text{ mA} \\
\text{R} & = 2k \Omega \\
\text{R} & = 2k \Omega \\
\text{R} & = 6k \Omega \\
\text{R} & = 8k \Omega
\end{align*}
\]

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**Other Equations Derived from Ohm’s Law**

\[
\begin{align*}
\text{I} & = \frac{V}{R} \\
\text{P} & = \frac{V^2}{R} \\
\text{power} & = \text{current} \times \text{voltage}
\end{align*}
\]

Recall that relationships between current and voltage are sign sensitive

- **Passive Sign Convention**
  - Current enters the positive terminal of an element
  - If the PSC is not satisfied:
    - \( \text{PSC satisfied: } \text{v} = \text{iR} \)
    - \( \text{PSC not satisfied: } \text{v} = -\text{iR} \)
  - Recall that relationships between current and voltage are sign sensitive

- **Short Circuit as Zero Resistance**
  - An element (or wire) with \( R = 0 \) is called a short circuit
  - Often just drawn as a wire (line)
  - Could draw a resistor with \( R = 0 \), but is unnecessary and adds clutter

\[
\begin{array}{c}
\text{Resistors cannot produce power} \\
\text{therefore, the power absorbed by a resistor will always be zero} \\
\text{1 \Omega = 1V / A}
\end{array}
\]
Open Circuit as Current Source (0 A)

The insulator (air) usually wins. Else, sparks fly.
In practice, you blow the current source (if not protected).
Frisable force meets immovable object.
Cannot connect a current source to an open circuit.
Could draw a source with $I = 0$, but is not done in practice.

As current source $I = 0$ A is also equivalent to an open circuit.

Short Circuit as Voltage Source (0 V)

The conductor usually wins. Else, sparks fly.
In practice, the wire usually wins and the voltage source melts (it's not protected).
Frisable force meets immovable object.
Cannot connect a voltage source to a short circuit.
Could draw a source with $V = 0$, but is not done in practice.

As voltage source $V = 0$ is also equivalent to a short circuit.
Nodes Defined

- **Node**: the point of connection between two or more branches.
- May include a portion of the circuit (more than a single point)

Example: How many nodes? How many essential nodes?

Essential Node: the point of connection between three or more branches

Example: How many essential nodes?

Branches Defined

- **Branch**: a single two-terminal element in a circuit.
- Segments of wire are not counted as elements (or branches)

Example: How many branches?

Example: How many loops?

Loops Defined

- **Loop**: any closed path in a circuit

Before we can begin analyzing a circuit, we need a common language and framework for describing circuits.
Kirchhoff's Current Law for Boundaries

\[ i_1 + i_3 = i_2 + i_4 \]

KCL also applies to closed boundaries for all circuits.

 Kirchhoff’s Current Law (KCL): the algebraic sum of currents entering a node (or a closed boundary) is zero.

The sum of currents entering a node is equal to the sum of the currents leaving a node.

Common sense:

All of the electrons have to go somewhere.

Based on law of conservation of charge.

They govern how elements within a circuit are related.

Kirchhoff’s laws tell us how the voltages and currents in different branches are related.

The defining equations for circuit elements (e.g., Ohm’s law).

The foundation of circuit analysis is

The defining equations for circuit elements (e.g., Ohm’s law).

• The foundation of circuit analysis is

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Comments on Ohm's Law, KCL, and KVL

Ohm's Law:
\[ v = \pm iR \]

KCL:
\[ \sum I_n = 0 \]

KVL:
\[ \sum V_m = 0 \]

Much of the circuit analysis that we will do is based on these three laws alone are sufficient to analyze many circuits.

Ohm's Law for Fluids

Ohm's law applies in fluid mechanics.

For turbulent flow, the pressure is related to the rate of flow squared - not analogous.

For laminar flow,
\[ Q = \pi r^2 \Delta P/8 \mu L \]

where
- \( Q \) = flow rate (\( m^3/s \))
- \( r \) = pipe radius (\( m \))
- \( L \) = pipe length (\( m \))
- \( \Delta P \) = pressure drop (\( kN/m^2 \))
- \( \mu \) = dynamic viscosity of fluid

Kirchhoff's Voltage Law (KVL)

Based on the conservation of energy. An analogous idea in hydraulic systems: sum of pressure drops and rises in any closed path must be equal.

KVL: the algebraic sum of voltages around a closed path (or loop) is zero.

Example 3: Kirchhoff's Voltage Law

Apply KVL to each loop in the circuit.

\[ 0 = \sum V \]

Kirchhoff's Voltage Law
Example 5: Applying the Basic Laws

\[ V_o = 4 \text{ V} \]
\[ R_1 = 6k \Omega \]
\[ R_2 = 4k \Omega \]

Find \( i_o \) and \( v_o \).

Ohm's Law for Fluids Continued

• If we define
  \[ R = \frac{8 \mu L}{\pi r^4} \]
  then
  \[ Q = \frac{\Delta P}{R} \]

• This is Ohm's law for laminar fluid flow in a pipe.

• Kirchhoff's laws also apply to fluid networks.

• Analogs
  - \( \text{Resistor} \leftrightarrow \text{Pipe} \)
  - \( \text{Voltage source} \leftrightarrow \text{Pressure source} \)
  - \( \text{Current source} \leftrightarrow \text{Flow rate source} \)
  - \( \text{Capacitor} \leftrightarrow \text{Fluid capacitance (tanks)} \)
  - \( \text{Inductor} \leftrightarrow \text{Fluid inductance (inertia)} \)
  - \( \text{Transformers} \leftrightarrow \text{Fluid transformers (change in pipe diameter)} \)

• But there are no fluid analogs to transistors or op amps.
Example 6: Applying the Basic Laws

Find $i_7$, $i_3$, $v_3$, and $v_I$.