Circuit Analysis: Introduction

- An electric circuit is an interconnection of electrical elements.
- Charge is an electrical property of the atomic particles which matter consists, measured in coulombs (C).
- 1 C of charge requires $6.24 \times 10^{18}$ electrons.
- Law of conservation of charge: charge cannot be created or destroyed, only transferred.

Basic Concepts Overview

- SI Prefixes
- SI Prefixes
- These are used throughout the curriculum and by engineers.

Circuit Analysis

- Flashlight Circuit
- We will use symbols for circuit elements.
- A flashlight circuit has 4 circuit elements.

Flashlight Circuit

- Multiplier Prefix Symbol Example
- Ideal Sources
- Active & Passive Elements
- The Borsig-law convention: Current, Voltage, Power, & Energy.
**Key Assumptions**

For the duration of this class, we will make the following assumptions:

- Propagation is instantaneous (lumped-parameter system)
- Net charge stored by all circuit elements is 0 (zero)
- Propagation is instantaneous (lumped-parameter system)
- Average drift velocity is very small (mm/sec)

### Electric Current

**Electric Current**

- Electric Current is the rate of change of charge, measured in amperes (A)
- Current in circuits physically realized by movement of electrons
- Electrons have a negative charge
- They move in the opposite direction of current

**Notes on Current**

- Current in circuits physically realized by movement of electrons
- Electrons have a negative charge
- By convention, current direction is defined as how positive charge moves

**Formal Definition**

\[ i = \frac{dq}{dt} \]

where

- \( i \) = current in amperes
- \( q \) = charge in coulombs
- \( t \) = time in seconds

**Symbols**

- \( \frac{dp}{dt} \) = \( i \)
Example 1: Voltage Concept

Voltage is the energy absorbed or expended as a unit charge moves through a circuit element. Analogous to pressure in a hydraulic system.

\[ v = \frac{w}{q} \]

where

- \( v \) = voltage in volts
- \( w \) = energy in Joules
- \( q \) = charge in coulombs

Example 2: Voltage Concept

Voltage is similar to the force due to gravity (mg).

Electric Current is analogous to marble velocity.

Viscosity is similar to resistance.

The light bulb (resistor) resists the flow of current. The current is the rate of flow of charge (i.e., electrons). The voltage sources push current through the circuit.

Voltage pushes charge in one direction. It is a measure of the potential between two points. It can be created by a separation of charge. Sometimes called potential difference.

Voltage moves through a circuit element. Voltage is the energy absorbed or expended as a unit charge.

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Voltage moves through a circuit element. Voltage is the energy absorbed or expended as a unit charge.
Passive Sign Convention

• Passive Sign Convention (PSC): Current enters the positive terminal of an element.
  - Equivalent: Current leaves the negative terminal.

• Most two-terminal circuit elements (e.g. batteries, light bulbs, resistors, switches) are characterized by a single equation that relates voltage to current:
  \[ v = \pm f(i) \]
  or
  \[ i = \pm g(v) \]

  • Called the defining equation.

  • The PSC determines the sign of the relationship:
    - If PSC is satisfied: \[ v = f(i) \]
    - If PSC is not satisfied: \[ v = -f(i) \]
    - If PSC is satisfied: \[ i = g(v) \]
    - If PSC is not satisfied: \[ i = -g(v) \]

  • This is also true of power:
    - If PSC is satisfied: \[ p = +v_i \]
    - If PSC is not satisfied: \[ p = -v_i \]

Energy

• Law of Conservation of Energy: the net power absorbed by some energy-absorbing element in a circuit is equal to the total energy absorbed by all energy-absorbing elements in the circuit.

• Energy: capacity to do work, measured in joules (J).

  \[ \text{Energy} = \int_0^t p(t) \, dt \]

  \[ \text{Energy} = \int_0^t v(t) \, i(t) \, dt \]

  \[ \text{Energy} = \int_0^t \, \text{Joules} \]

  \[ \text{Power} = \frac{\text{Energy}}{\text{time}} \]

Example 3: Passive Sign Convention

Suppose the circuit element shown below is characterized by:

\[ v = \pm f(i) \]

and

\[ i = \pm g(v) \]

Determine whether the PSC is satisfied and write the equations for the voltage, current, and power for each of the diagrams below.

\[ \begin{align*}
  v & = \pm f(i) \\
  i & = \pm g(v)
\end{align*} \]
Passive Sign Convention Remarks

- In most cases, we will be concerned with voltage drops.
- The first expression is the most common:
  - There is a $-5 \text{ V}$ drop from $b$ to $a$.
  - There is a $-5 \text{ V}$ rise from $a$ to $b$.
  - There is a $+5 \text{ V}$ rise from $b$ to $a$.
  - There is a $+5 \text{ V}$ drop from $a$ to $b$.
- The following statements are true and equivalent:

Voltage Drops & Rises Defined

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  - There is a $+5 \text{ V}$ drop from $b$ to $a$.
- The first expression is the most common.
- In most cases, we will be concerned with voltage drops.

Example 4: Passive Sign Convention

Find the total power absorbed in the circuit.

Voltage Current Power (W) (A)

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<tr>
<th>v</th>
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Example 5: Passive Sign Convention

Find the total power absorbed in the circuit.

Voltage Current Power (W) (A)

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Notes on Ideal Sources

- Ideal sources are models used to simplify analysis.
- These devices do not exist physically.
- How much power can an ideal source produce?
  - How much power can a battery produce?
- Ideal models serve as a good approximation of physical devices.
  - Ideal models do not exist physically.
- Ideal sources are models used to simplify analysis.

**Ideal Voltage Sources**

- Ideal Voltage Source: Produces $V_s$ volts regardless of the current absorbed or produced by the device.
- The voltage produced may depend on some other circuit variable (current or voltage).
- Note the sign of $V_s$ can be negative: $12\, V = -12\, V$.

**Ideal Current Sources**

- Ideal Current Source: Produces $I_s$ amps regardless of the current in the device.
- The current produced may depend on some other circuit variable.
- Note the sign of $I_s$ can be negative: $2\, A = -2\, A$.