Concrete Mixing EAS 199B
In-class Problem

A batch of masonry cement is made by mixing three 5-gallon buckets of wet sand with one 70 lb_f bag of masonry mix and 3 gallons of water. The sand used in the mix is partially wet, containing an unknown amount of water.

a. If the final batch of "mud" contains 11.3% water by weight when the desired consistency is reached, how many gallons of water did the sand used in the mixture contain? Assume the wet sand and dry sand occupy the same volume. Answer: 1.48 gal.
b. What is the weight of the final batch of mud?

Be sure to follow the systematic presented in class when solving this problem. Remember that learning the correct solution process is just as important as getting the correct answer.

Potentially useful information for this problem:

Specific weight of water = 8.33 lb_f/gal.

Specific weight of dry sand = 111 lb_f/ft^3.

1 gal = 0.1337 ft^3

Initial:

3 5-gallon pails of wet sand with unknown water content

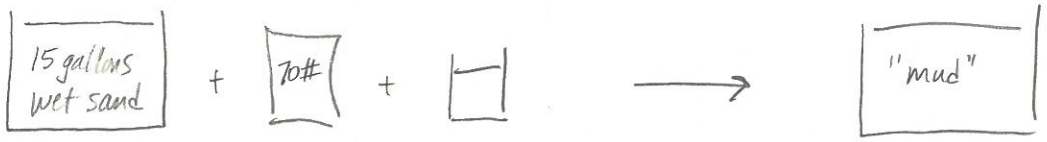
70 lb bag of masonry mix

3 gallons of water

Final: Unknown mass of "mud" made by mixing all the ingredients from the initial state. The final mixture is 11.3% H_2O

(1) This is a batch problem. The initial and final states are known and there is no flow.

(2) Schematic



$X_{w,i} = ?$
$X_{s,i} = ?$
$X_{w,i} + X_{s,i} = 1$

70 lb_f dry masonry mix

$W_{m,i} = 70\ lb_f$

3 gal H_2O
$m_{w,add} = \rho V$

$X_{w,f} = 0.113$
$X_{s,f} = ?$
$X_{m,f} = ?$

Note: The wet sand is the product of another mixing process

15 gal. dry sand
$\gamma_s = 111 \frac{lb_f}{ft^3}$

$W_w = ?$

wet sand
$W_s + W_w$

The water fills the voids in the dry sand without changing its volume

From the wet sand mixture we know that

$$W_{s,i} = \gamma v$$

$$= \left(111 \frac{lb_f}{ft^3}\right)\left(15\ gal\right)\left(\frac{0.1337\ ft^3}{gal}\right) = 222.6\ lb_f$$

$$\boxed{W_{s,i} = 222.6\ lb_f} = W_{dry,sand}$$

We can also calculate the weight of the three gallons of added water

$$W_{w,add} = \gamma_w v \qquad \gamma_w = 8.33\ lb_f/gal$$

$$= \left(8.33 \frac{lb_f}{gal}\right)\left(3\ gal\right) = \boxed{24.99\ lb_f = W_{w,add}}$$

(3) Write masses of each constituent at the initial and final state

	Initial		Final	
water	$W_{ws,i} X_{w,i} + W_{w,add}$	$=$	$W_{mud} X_{w,f}$	(1)
sand	$W_{dry\,sand}$	$=$	$W_{mud} X_{s,f}$	(2)
dry masonry mix	$W_{dry,mix}$	$=$	$W_{mud} X_{m,f}$	(3)
Total	$W_{dry,sand} + W_{dry,mix} + W_{ws,i} X_{w,i} + W_{w,add}$	$=$	W_{mud}	(4)

Unknowns: $W_{ws,i}$, $X_{w,i}$, W_{mud}, $X_{s,f}$ $\quad (X_{m,f} = 1 - X_{w,f} - X_{s,f})$

Four equations

Combine Equation (1) and (4)

$$W_{ws,i} X_{w,i} + W_{w,add} = \left(W_{dry,sand} + W_{dry,mix} + W_{ws,i} X_{w,i} + W_{w,add}\right) X_{w,f}$$

group $W_{s,i} X_{w,i}$

$$W_{ws,i} X_{w,i} \left[1 - X_{w,f}\right] = \left(W_{dry,sand} + W_{dry,mix} + W_{w,add}\right) X_{w,f} - W_{w,add}$$

Substitute known values to simplify

$X_{w,f} = 0.113$

$W_{drysand} = 222.6\ lb_f$

$W_{dry\,mix} = 70\ lb_f$

$W_{w,add} = 24.99\ lb_f$

$317.59\ lb_f$

$$\Rightarrow W_{ws,i} X_{w,i} \left(1 - 0.113\right) = \underbrace{\left(317.59\ lb_f\right)\left(0.113\right) - 24.99\ lb_f}_{10.90\ lb_f}$$

$$\therefore W_{w,s} X_{w,i} = \boxed{12.29\ lb_f = W_{w,i}}$$

Substitute this result back into Equation (1)

$$W_{mud} = \frac{1}{X_{w,f}} \left[W_{ws,i} X_{w,i} + W_{w,add}\right]$$

$$= \frac{1}{0.113} \left[12.29 lb_f + 24.99\ lb_f\right] = 329.88$$

$$\boxed{W_{mud} = 329.9\ lb_f}$$

Compute the volume of water in the initial wet sand

$$W_{w,i} = \gamma_w V_w \Rightarrow V_w = \frac{W_{w,i}}{\gamma_w} = \frac{12.29\ lb_f}{8.33\ lb_f/gal}$$

$$\boxed{V_w = 1.48\ gal}$$

CHECK: Use Equations (2) and (3) to compute $X_{s,f}$ and $X_{m,f}$ and then test whether $X_{w,f} + X_{s,f} + X_{m,f} = 1$

Equation (2) $\Rightarrow$ $X_{s,f} = \frac{W_{dry\ sand}}{W_{mud}} = \frac{222.6\ lb_f}{329.9\ lb_f} = 0.6749$

Equation (3) $\Rightarrow$ $X_{m,f} = \frac{W_{dry\ mix}}{W_{mud}} = \frac{70\ lb_f}{329.9\ lb_f} = 0.2122$

Now test:

$$X_{w,f} + X_{s,f} + X_{m,f} \stackrel{?}{=} 1$$

$$0.113 + 0.6749 + 0.2122 \stackrel{?}{=} 1$$

$$0.9999 \stackrel{?}{=} 1 \quad \checkmark$$

Close enough given the rounding errors in the intermediate calculations