Correction Control of Fish Tank Salinity — EAS 199B
In-class Problem

Assume that your fishtank system has a setpoint of 0.09% NaCl. Your instructor comes by your table and upsets your system by adding a good dose of DI water. The conductivity circuit returns an analog output that corresponds to a salinity of 0.04% NaCl (which is below LCL).

a. What is the target concentration if you have a gain of 0.80 (80%)?
b. Using this gain, how much salty water (1% NaCl) should be added?
c. How long should you leave the valve open if the flow rate is 0.2L/min?

Recommended assumptions:

1. The water leaves at the overflow is a mixture of water from the salty tank and the fishtank.
2. The most salty the overflow water can be is 1% NaCl, and the least salty it can be is 0.04% NaCl. Assume that 15% of the overflow water is 1% NaCl and that the rest is 0.04% NaCl.
3. Neglect density differences between incoming and outgoing water; that is, the mass of water that comes in from the salty tank is equal to the mass of water that leaves through the overflow.

Initial Observations

Set point = 0.09% NaCl $\Rightarrow X_s = 0.0009$ the <u>desired</u> final state

After the disturbance (addition of DI water) $X_s = 0.0004$

That is the initial condition for our analysis

a.) What is the target salinity (mass fraction) if the gain is 0.8?

$$\text{Error} = \underset{\substack{\uparrow \\ \text{desired}}}{0.0009} - \underset{\substack{\uparrow \\ \text{current}}}{0.0004} = 0.0005$$ error in mass fraction

$$\text{gain} * \text{Error} = (0.8)(0.0005) = 0.0004$$ = correction in mass fraction achieved if the gain is 0.8

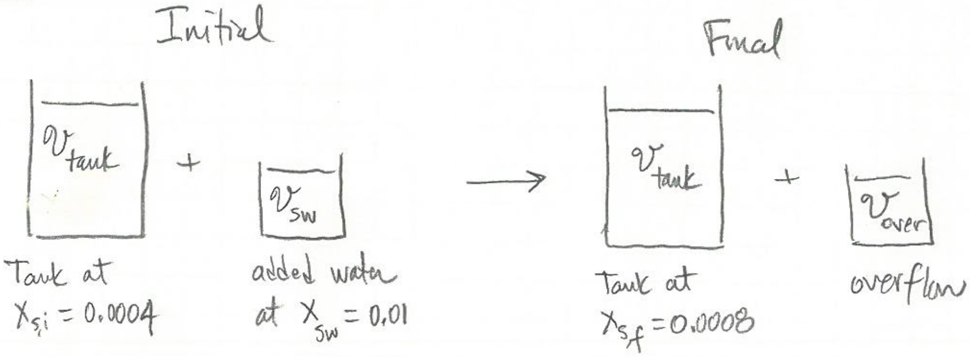
$$\text{target} = X_{s,i} + \text{gain} * \text{error}$$

$$= 0.0004 + 0.0004 = \boxed{0.0008 = X_{s,f}}$$

Final mass fraction if a gain of 0.8 is applied to the error <u>and</u> initial mass fraction is 0.0004

b.) How much salty water (1% NaCl) should be added to achieve the desired end state corresponding to a gain of 0.8? This end state has a final mass fraction of salt of $X_{s,f} = 0.0008$. (solution to part a.)

Schematic



This can be treated as a batch problem.

Known:

$\forall_{tank} = \frac{\pi}{4} d^2 h$ take $d = 1.6$ inch $=$ I.D. of PVC pipe

$h = 2.0$ inch $=$ assumed dept of water

$$= \frac{\pi}{4}\left(1.6\text{ in} * \frac{2.54\text{ cm}}{\text{in}}\right)^2\left(2.0\text{ in} * \frac{2.54\text{ cm}}{\text{in}}\right)$$

$$\forall_{tank} = 65.9\text{ cm}^3 * \frac{1\text{ L}}{1000\text{ cm}^3}$$

$$\forall_{tank} = 0.0659\text{ L}$$

Assume $\rho_w = 1.0 \frac{kg}{L}$ The problem statement says to ignore the effect of salinity on density

$$\Rightarrow M_{w,tank} = \rho_w \forall_{tank} = 0.0659\text{ kg}$$

$$\boxed{M_{w,tank} = 65.9\text{ g}}$$

= mass of mixture in the tank because we are neglecting the effect of salinity on density

Mass fraction of water in overflow can be computed from information in the problem statement

overflow is 15% water with $X_s = 0.01$

and 85% water with $X_s = 0.0004$

$$\Rightarrow X_{s,overflow} = (0.15)(0.01) + (0.85)(0.0004) = 0.0018$$

$$X_{w,overflow} = 1 - X_{s,overflow} = 0.9982$$

Mass balances for constituents and overall

Initial = Final

water: $M_{w,t} x_{w,i} + M_{w,add} X_{w,add} = M_{w,t} X_{w,f} + M_{over} X_{w,over}$ (1)

salt: $\check{M}_{w,t} \check{X}_{s,i} + M_{w,add} \check{X}_{s,add} = \check{M}_{w,t} \check{X}_{s,f} + M_{over} \check{X}_{s,over}$ (2)

total: $M_{w,t} + M_{w,add} = M_{w,t} + M_{over}$ (3)

Equation (3) $\Rightarrow$ $M_{w,add} = M_{over}$ This follows from the requirement (assumption) that the water level in the tank doesn't change and that the density of the saltwater mixture does not depend on salinity

In Equation (2) we know everything except $M_{w,add}$ $(= M_{over})$

Solve Equation (2) for $M_{w,add}$

$$M_{w,t} X_{s,i} + M_{w,add} X_{s,add} = M_{w,t} X_{s,f} + M_{w,add} X_{s,over}$$

$$M_{w,add} \left(X_{s,add} - X_{s,over} \right) = M_{w,t} \left(X_{s,f} - X_{s,i} \right)$$

$$M_{w,add} = M_{w,t} \frac{X_{s,f} - X_{s,i}}{X_{s,add} - X_{s,over}}$$

$$= (65.9\ g) \frac{(0.0008 - 0.0004)}{(0.1000 - 0.0018}$$

$$\boxed{M_{w,add} = 3.21 g}$$

c.) How long should the valve be open if the desired $m_{w,add}$ is added when the flow rate is $Q = 0.2\ L/min$?

$$m_{w,add} = \rho_w V_{add} \Rightarrow V_{add} = \frac{m_{w,add}}{\rho_w} = \frac{3.21\ g}{1000\ g/L} = 3.21 \times 10^{-3}\ L$$

$$V_{add} = 3.21 \times 10^{-3}\ L \qquad Q = 0.2\ L/min$$

$$Q = \frac{V_{add}}{\Delta t} \Rightarrow \Delta t = \frac{V_{add}}{Q} = \frac{3.21 \times 10^{-3}\ L}{0.2\ L/min} = 0.0161\ min$$

$$\boxed{\Delta t = 0.0161\ min = 0.96\ s}$$