Propeller (Axial−flow) Pumps

- Rigorous design analysis based on energy−momentum principles is not available
- Operation can be described by principle of ***impulse−momentum***
- Linear impulse is defined as integral of product of force vs. time *dt* from *t*’ to *t*”

$$I = \int_{t'}^{t''} F dt$$

- If a constant force is involved during time T, the impulse simplifies to

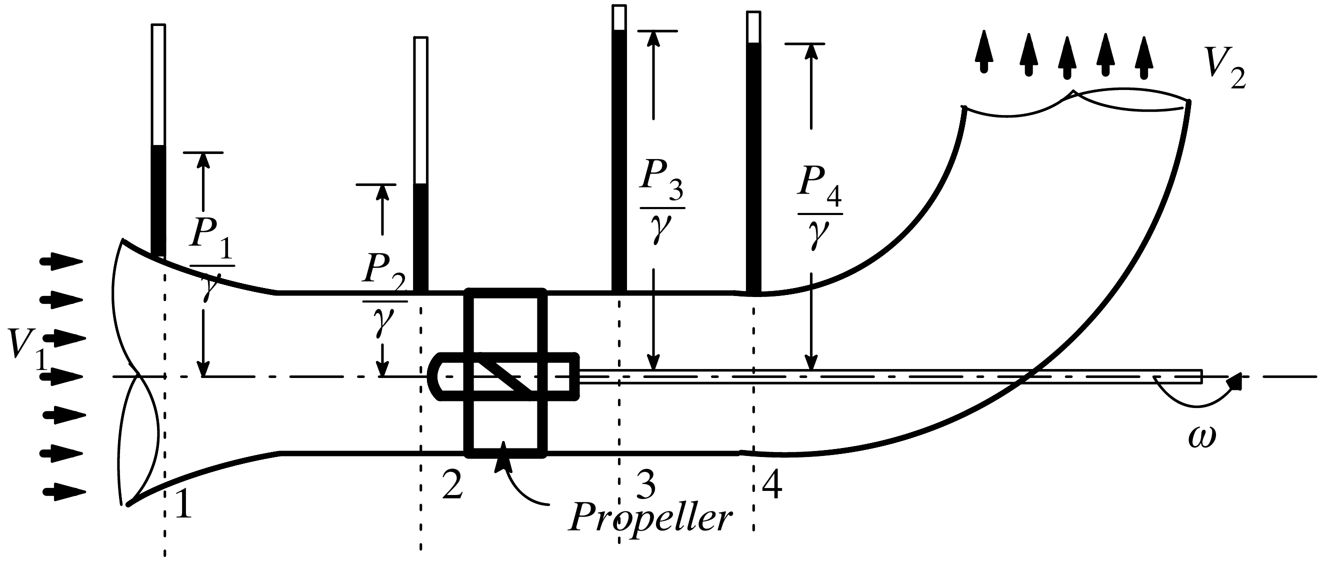
$$(impluse) = (force)\cdot(time)$$

- Impulse−momentum requires that force acting on system be equal to change in linear momentum

$$(force)\cdot(time) = (mass)\cdot(velocity\ change)$$

or

$$(force) = \frac{(mass)\cdot(velocity\ change)}{(time)} \quad \boldsymbol{A}$$



Propeller Pump

- Applying to control section in pump

$$\frac{(mass)}{(time)} = \frac{(density)(volume)}{(time)} = (density)(discharge) = \rho Q$$

- The velocity change between the two ends of the control volume

$$(velocity\ change) = V_i - V_f$$

- Substituting into A

$$\sum F = (force) = \rho Q(V_i - V_f)$$

- As fluids moves from 1 – 2, velocity increases, pressure drops

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

- From 2 –3, energy is added by propeller as pressure head
- From 3–4, slight head loss and increase velocity (taper in pump)
- Applying impulse momentum between 1–4

$$P_1A_1 + F - P_4A_4 = \rho Q(V_4 - V_1)$$

  - ◆F is force of propeller on fluid

- If conduit is uniform diameter, $V_1 = V_4$ so

$$F = (P_1 - P_4)A$$

  ◆Pump is used to generate pressure

- If $V_2 = V_4$, then

$$\frac{P_3 - P_2}{\gamma} = \left(\frac{P_4}{\gamma} + \frac{V_4^2}{2g}\right) - \left(\frac{P_1}{\gamma} + \frac{V_1^2}{2g}\right) = H_P$$

- Total power output of the pump is

$$P_o = \gamma Q H_P = Q(P_3 - P_2)$$

- Propeller pumps are used for low head (<12 m) and high capacity (> 20 L/s)

- More than one set of propeller can be mounted on one axis to form ***multistage propeller pump***

  ◆High Q, High head

  ◆Self–priming, used in deep water wells

Example: A 3 m diameter pump delivers a large Q over 2.6 m elevation head. If shaft power to pump (e = 80%) is 1500 kw, what is Q?

Solution

Energy imparted to pump

$$P_o = e_p P_i = 0.8 \cdot 1500\,kw = 1{,}200{,}000\,w$$

and

$$P_0 = \gamma Q H_p = \gamma Q\left(h + \sum k\frac{V^2}{2g}\right)$$

Assuming $k_{in} = 0.5$ and $k_{out} = 1.0$

$$P_0 = \gamma Q\left(h + 1.5\frac{Q^2}{2gA^2}\right); \quad \gamma = 9810\,N/m^3$$

and

$$1,200,000 = 9810\,Q\Big(2.6 + 1.5\frac{Q^2}{(1.5\pi)^2 \cdot 2 \cdot 9.81}\Big)$$

Trial & error

$$Q = 30.45\ m^3/s$$

Jet (Mixed−flow) Pumps

- Utilize a high pressure stream to transfer momentum
- Often used in combination with centrifugal pumps
- Compact & light no moving parts
- High energy loss
- Used in construction for dewatering no moving parts
- See figure 5.7

Selection of A Pump

- Efficiency of pump depends on discharge, head, and power of pump
- Approximate application ranges are given in Figure 5.8
- For a given pipeline, an H−Q curve is plotted
- H−Q curve is matched to pump performance chart
- See figure 5.9 & 5. 10

Example

A pump delivers 70 L/s of water between two reservoirs 1000 m apart with elevation difference of 20 m with 20 cm commercial steel pipe. Determine pump based on figures 5.9 & 5.10

Solution

Steel pipe roughness e = 0.045 mm. Pipe velocity

$$V = \frac{Q}{A} = \frac{0.070\, m^3/s}{\left(\frac{\pi}{4}\cdot 0.2^2\right) m^2} = 2.23\, m/s \;\; R_e = \frac{VD}{\nu} = \frac{2.23\cdot 0.2}{1\cdot 10^{-6}\, m^{2/s}} = 4.5\cdot 10^5$$

and

$$\frac{e}{D} = 0.045mm/200mm = 2.3\cdot 10^{-4} = 0.00023$$

From Moody $f = 0.016$

Pipe friction loss is

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.016\cdot\frac{1000}{0.2}\cdot\frac{2.23^2}{2\cdot 9.81} = 20.27\, m$$

Ignoring minor losses, the pump must work against

$$H_P = (elevation\ difference) + (friction\ loss) = 20 + 20.27 = 40.27\, m$$

- From Manufacture’s figure 5.9 – choose pumps II or III
- Usually pumps operate over range of H vs. Q
- Now develop H−Q curve

$Q(L/s)$	$V(m/s)$	R_e	f	H_f	H_P
50	1.59	$3.2 \cdot 10^5$	0.0165	10.65	30.65
60	1.91	$3.8 \cdot 10^5$	0.016	14.87	34.87
80	2.55	$5.1 \cdot 10^5$	0.0155	25.61	45.61

Now matching with 5.10

First Trial: Pump II at 4350 rpm

$$Q = 70\, L/s \;\; H_P = 40.3\, m \quad \therefore\; P_i \approx 71\, hp \;\text{ and }\; e \approx 52\%$$

Second Trial: Pump III at 3850 rpm

$$Q = 68\, L/s \;\; H_P = 39.0\, m \quad \therefore\; P_i \approx 61\, hp \;\text{ and }\; e \approx 57\%$$

Third Trial: Pump III at 4050 rpm

$$Q = 73\,L/s \;\; H_P = 42\,m \quad \therefore \; P_i = 70\,hp \;\; \text{and} \;\; e = 58\%$$

- Which to choose?
  - ◆Hydraulically: pump II @ 4350 rpm – best fit
  - ◆Pump III @ 3850 rpm & 4050 rpm are close
  - ◆Look at cost of pump vs. energy cost