Topics covered by this lecture:

1. Steady Uniform Flow In Open Channels

Steady Uniform Flow in Open Channels

Definition: ***Uniform flow*** is when no change in velocity occurs along the channel

- Flow streamlines are straight and parallel
- Velocity doesn't change, then velocity head is constant

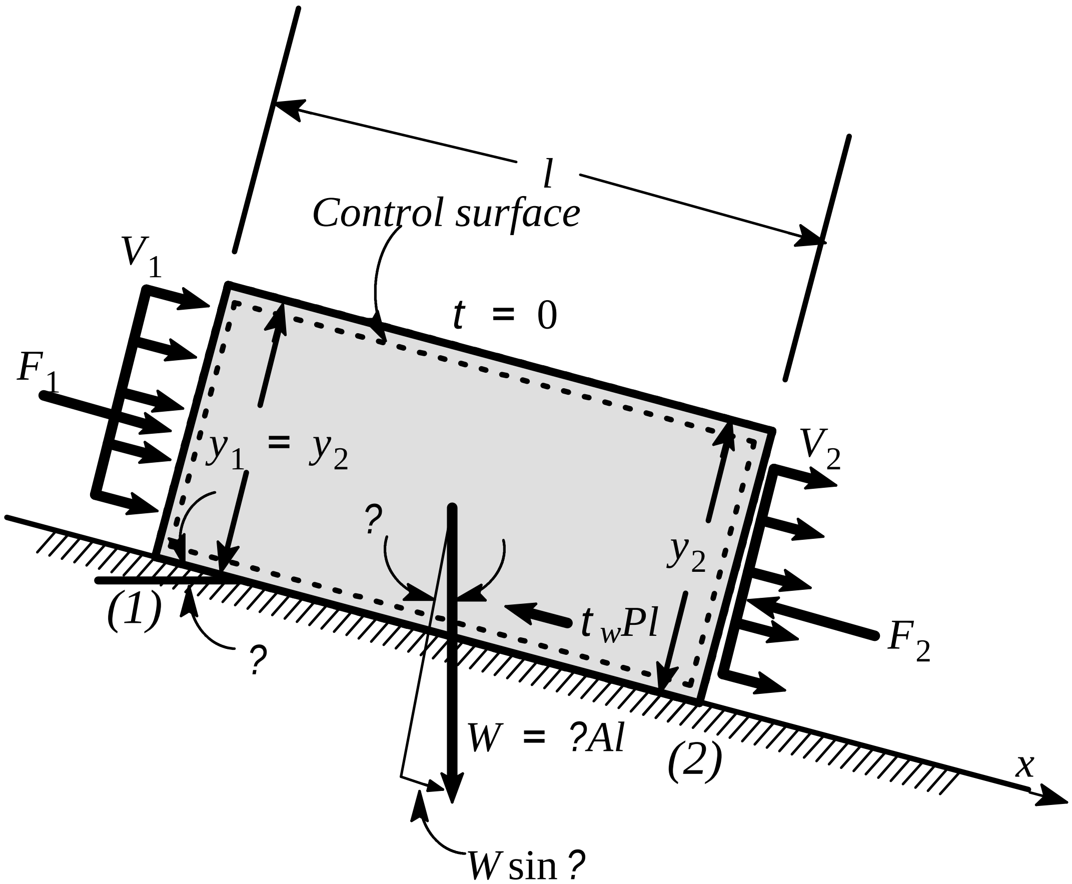
Uniform flow conditions:

1. Channel must be straight
2. No change in slope
3. No change in cross- section

- Called a ***prismatic*** channel

Definition: when flow is uniform, *depth* is called ***normal depth***

Using the definition diagram shown below



Control Volume for Uniform Flow in Open Channel

Assuming steady uniform flow, the *x* component of the momentum equation applied to the above control volume yields

$$? \; F_x = \tilde{A}Q(V_2 - V_1) = 0$$

S $V_1 = V_2$, no acceleration of fluid

Writing the balance of forces

$$F_1 - F_2 - t_w Pl + W \sin ? = 0$$

S Because $y_1 = y_2$, $F_1 = F_2$

- S W sin ? = component of fluid weight acting down slope (motive force)

- S t_wPl = shear force acting upslope because of interaction of water and wetted perimeter (resisting force)

Thus, the force equation becomes

$$t_w = \frac{W\sin ?}{Pl} = \frac{WS_0}{Pl}$$

If we assume sin ? ˜ tan ? = S_0 , since the bottom slope is small (S_0 << 1).

Since W = ?Al and the hydraulic radius R_h = A/P, the equation becomes

$$t_w = \frac{?AlS_0}{Pl} = ?R_hS_0$$

- S Most open channel flows are turbulent (very large Re)

- S Friction factor *f* is independent of Re for large Re

- S In this case, shear stress is proportional to dynamic forces (V^2/2g) and independent of viscosity, or

$$t_w = K\frac{V^2}{2g}$$

where K depends on the roughness of the channel.

Equating these two expressions

$$K\frac{V^2}{2g} = ?R_hS_0$$

$$V = C ?\overline{R_hS_0}$$

- C is called the ***Chezy coefficient*** after A. Chezy
- Equation is called ***Chezy equation***

Irish Engineer Manning improved equation, to more accurately describe the dependence on R_h as

$$V = \frac{k}{n} R_h^{2/3} S_0^{1/2}$$

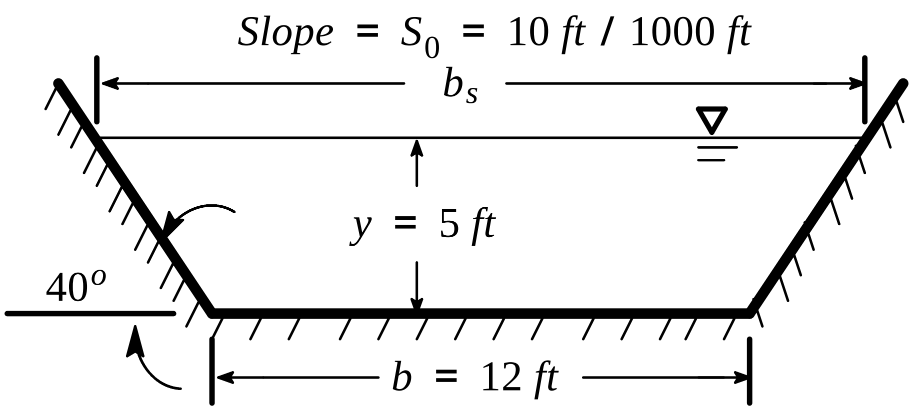
$$Q = \frac{k}{n} AR_h^{2/3} S_0^{1/2}$$

This is known as the ***manning equation***, and *n* is the ***manning co-efficient***.

- k = 1 if SI units are used
- k = 1.49 if English units are used.
- n ranges from .01 - .15 depending on material
  - ✎See tables in text

Channel Surface	**n- value**
Gravel Beds	0.025 - 0.040
Earth	0.026 - 0.050
Cement	0.011 - 0.016
Wood	0.012 - 0.013
Asphalt	0.013 - 0.016

Example: Water flows in the canal of trapezoidal cross section as shown in the figure below. The bottom drops 10 ft per 1000 ft of length. Determine the flowrate for new smooth concrete or a weedy channel.



Uniform Flow in Trapezoidal Channel

From the Manning equation

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2}$$

For the depth of 5 ft, the area A of flow is

$$A = 12\,ft(5\,ft) + 5\,ft\left(\frac{5}{\tan 40^o}\,ft\right) = 89.8\,ft^2$$

The wetted perimeter P of flow is

$$P = 12\,ft + 2\left(\frac{5\,ft}{\sin 40^o}\right) = 27.6\,ft$$

Therefor the hydraulic radius R_h is

$$R_h = \frac{A}{P} = \frac{89.9\,ft^2}{27.6\,ft} = 3.25\,ft$$

- Note with the free surface width b_s = 23.9 ft, the hydraulic radius is less than one depth!

With S_0 = 10 ft/1000 ft = 0.01, Manning’s equation becomes

$$Q = \frac{1.49}{n}(89.8)(3.25)^{2/3}(0.01)^{1/2} = \frac{29.37}{n}$$

where Q is in ft^3/s.

- For smooth concrete n = 0.012 flowrate Q_s

$$Q_s = \frac{29.37}{0.012} = 2448\ cfs$$

- For weedy conditions n = 0.030flowrate Q_w

$$Q_w = \frac{29.37}{0.03} = 979\ cfs$$

- Corps of Engineers go hog- wild on this one!

The corresponding velocities are

$$V_s = \frac{Q_s}{A} = 27.25\ ft/s$$

$$V_w = \frac{Q_w}{A} = 10.9\ ft/s$$

- Note the roughness causes a decrease in flowrate
- Note that for 50 ^oF water

$$\mathrm{Re}_w = \frac{R_h V_w}{?} = \frac{(3.25\ ft)(10.9\ ft/s)}{1.41 \times 10^{-5}\ ft^{2/s}} \sim 3 \times 10^6$$

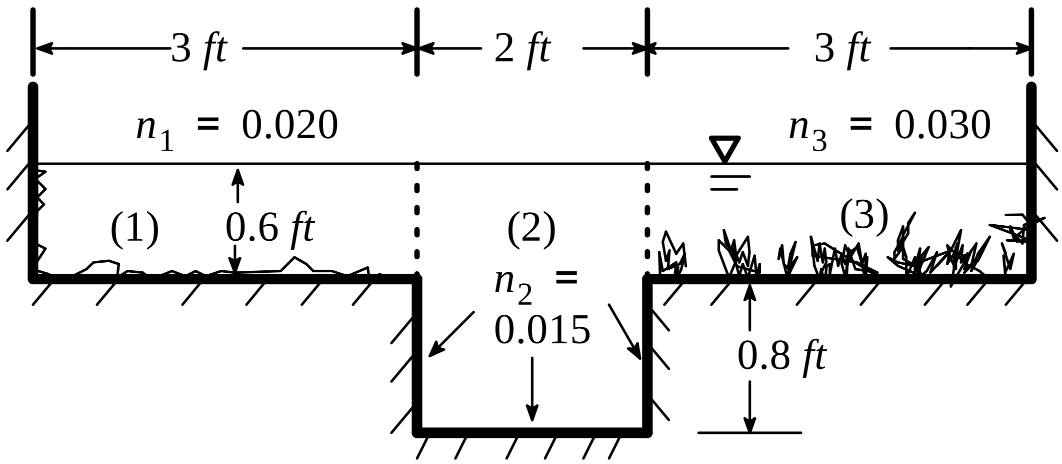
✎Well into turbulent range for slow velocity

Computing the Froude numbers based on maximum depth in channel

$$Fr_s = \frac{V_s}{?\overline{gy}} = \frac{27.25}{?\overline{(32.2\ ft/s^2)(5\ ft)}} = 2.15$$

$$Fr_w = \frac{V_w}{?\overline{gy}} = \frac{10.9}{?\overline{(32.2\ ft/s^2)(5\ ft)}} = 0.86$$

Example: Water flows along the drainage canal having the properties as shown below. If the bottom slope is 1 ft/ 500 ft, estimate the flowrate Q.



Uniform Flow in Weird Channel

Dividing the cross section into three sections (1), (2), (3), and writing the flowrate Q = $Q_1 + Q_2 + Q_3$

$$Q_i = \frac{1.49}{n_i} A_i R_{hi}^{2/3} S_0^{1/2}$$

The appropriate values for A_i, P_i, R_{hi}, n_i are listed below.

Table of Values for Weird Channel				
	A_i	P_i	R_{hi}	
i	(ft^2)	(ft)	(ft)	n_i
1	1.8	3.6	0.500	0.020
2	2.8	3.6	0.778	0.015
3	1.8	3.6	0.500	0.030

- Note portions of the perimeters between sections (indicated by dashed lines) are not included in P_i

$$A_2 = 2\,ft\,(0.8 + 0.6)\,ft = 2.8\,ft^2$$

and

$$P_2 = 2\,ft + 2(0.8\,ft) = 3.6\,ft$$

So

$$R_{h2} = \frac{A_2}{P_2} = \frac{2.8\,ft^2}{3.6\,ft} = 0.778\,ft$$

Thus the total flowrate is

$$Q = Q_1 + Q_2 + Q_3$$

$$= 1.49(0.002)^{1/2} \times \left[\frac{1.8(0.500)^{2/3}}{0.020} + \frac{2.8(0.778)^{2/3}}{0.015} + \frac{1.8(0.500)^{2/3}}{0.030}\right]$$

$$Q = 16.8\,cfs$$

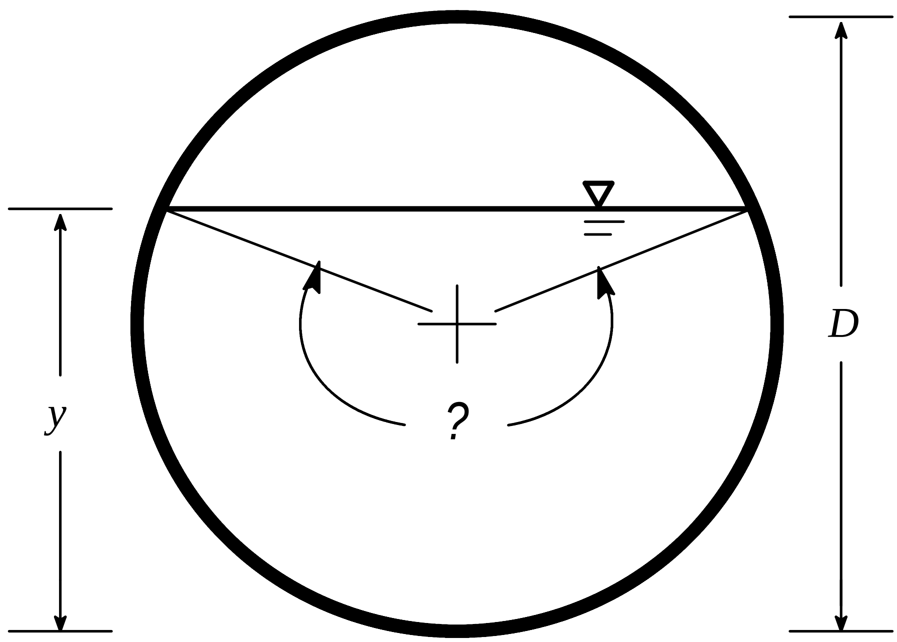
Flow in Conduits of Circular Cross- Section

- Examples: highway culverts, city sewers
- Hold slope and roughness constant in Manning's equation

$$Q = \frac{k}{n} A R_h^{2/3} S_0^{1/2}$$

- Flowrate Q is proportional to $AR_h^{2/3}$

Example: Determine flowrate for a circular pipe in uniform open channel flow conditions



Uniform Open Channel Pipe Flow

- Here flow depth y = *f*(?)

According to Manning's formula

$$Q = \frac{k}{n} A R_h^{2/3} S_0^{1/2}$$

Where k, *n* and S_o are constants. It can be shown from geometry that (Table 10.1 Fox)

$$A = \frac{D^2}{8}(? - \sin ?) \quad P = \frac{D?}{2} \quad R_h = \frac{D(? - \sin ?)}{4?}$$

where ? (the angle created by the flow) is in radians.

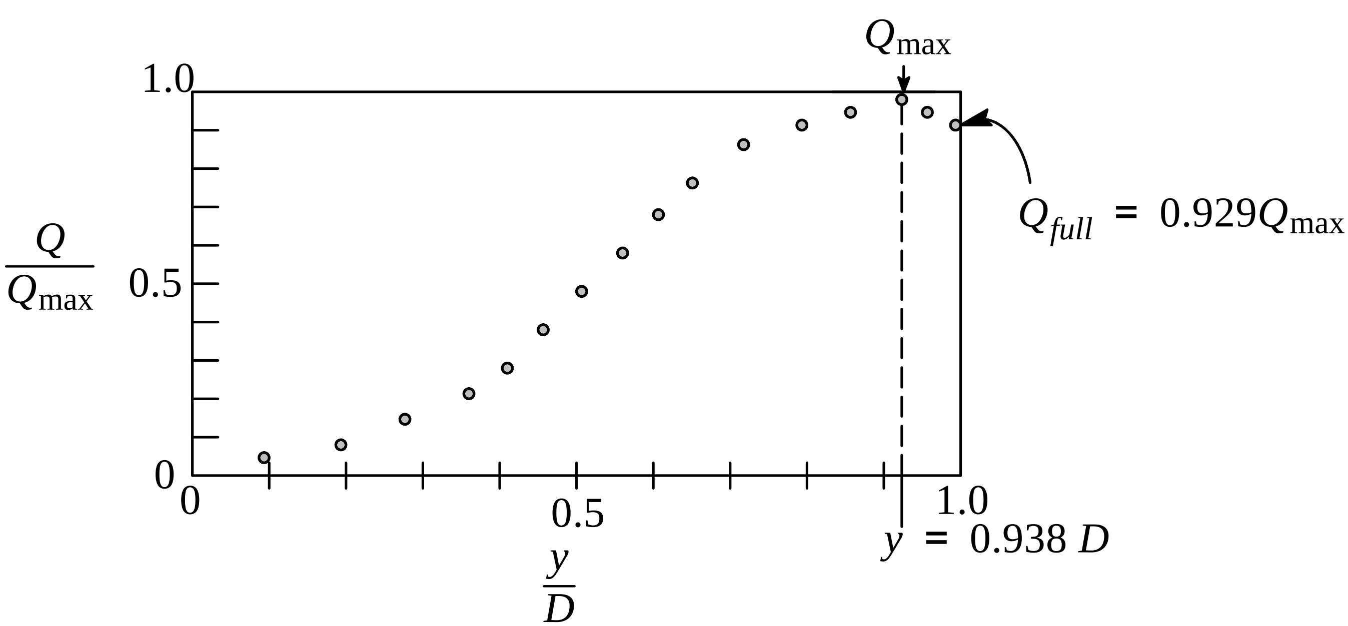
Manning's equation becomes

$$Q = \frac{k}{n} S_0^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \left[\frac{(? - \sin ?)^{5/3}}{?^{2/3}}\right]$$

The depth of flow y (as a function of pipe diameter D and and angle ?) is

$$y = \frac{D}{2}\left[1 - \cos\left(\frac{?}{2}\right)\right]$$

A graph of flowrate Q as function of depth y, Q = Q(y) is shown below



Normalized Flowrate vs. Normalize Depth for Uniform Open Circular Channels

- Note two depths possible for same Q
- Slight difference in maximum Q negligible in most practical problems, other errors (e.g. *n*) greater