Complexity Theory

easy		hard
$O(n^k)$	TSP	$\Omega(2^{n^k})$
sorting	Longest path	chess
rod cutting	Sudoku	finding all subsets of a set
shortest path	3SAT	halting problem
convex hull	Hamiltonian Cycle	
	graph coloring	
	protein folding	

Cook-Levin Theorem

Decision Problems

problems with a yes / no answer

Allows us to view problems as language recognition.

Encodings

- Since for this topic all polynomial behavior is the same, any two encodings that differ by only a polynomial are equally good.

Complexity Classes

P

- informally: the set of all decision problems we can efficiently solve
- The set of all decision problems that have polytime algorithms

$$P = \bigcup_{k \geq 0} O(n^k)$$

NP

- The set of all decision problems that can be solved in poly-time with a non-deterministic Turing Machine.
- The set of all decision problems that can be verified in poly-time.

P vs NP

Obviously $P \subseteq NP$

- We do not know if this inclusion is proper

$$P \subsetneq NP$$

or

$$P = NP$$

NP-Complete (NPC)

- The hardest problems in NP
- A solution to any NP-Complete problem can be used as a ~~start~~ solution to any NP problem.

Polynomial Reduction

- to reduce problem A to problem B means that any instance of problem A can be transformed into an instance of problem B

$$A \leq_p B$$

An algorithm for B can be used to solve A.

NPC Def

A language A is NP-Complete iff

1.) $\forall L \in NP \quad L \leq_p A$ ← this means A is NP-Hard

2.) $A \in NP$

Cook-Levin-Theorem

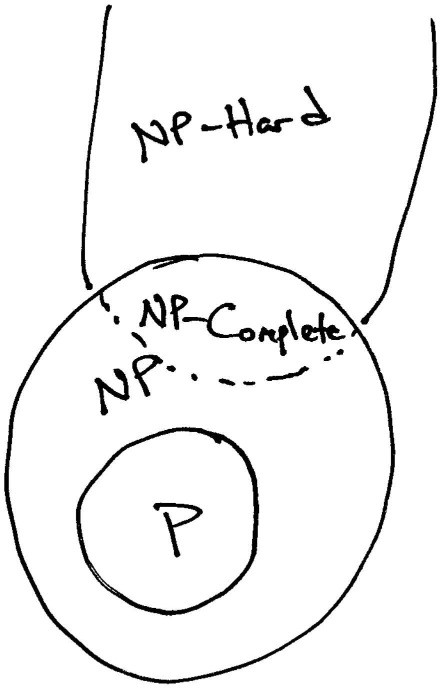
3SAT is NP-Complete

- the language of boolean formulas can be used to represent any language

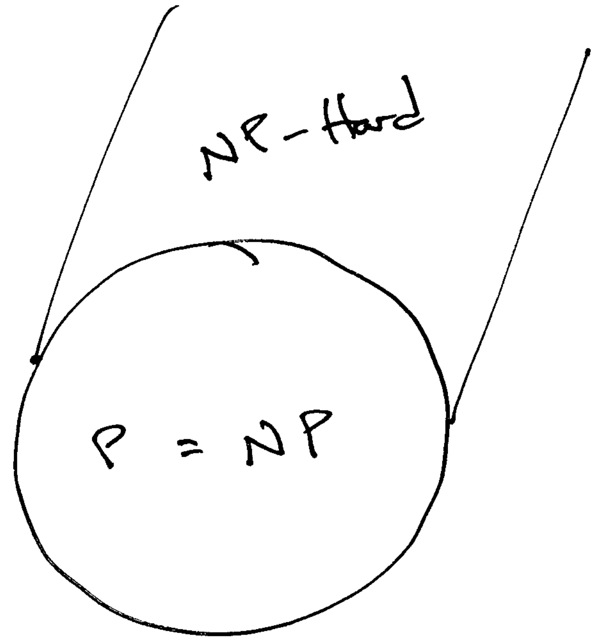
Polynomial Reductions are Transitive

if $A \leq_p B$ and $B \leq_p C$ then $A \leq_p C$

$P \subsetneq NP$



$P = NP$



$$P \subseteq NP \subseteq PSPACE \subseteq EXP$$

$$P \subsetneq EXP$$

Hamiltonian Cycle $\leq$ TSP

HC

Given an undirected graph, does it contain a Hamiltonian Cycle? i.e. a simple cycle that visits all vertices.

TSP

Given a weighted complete graph and a threshold T does the graph contain a Hamiltonian Cycle of length $\leq T$.

HC $\leq_p$ TSP

- Given an instance of HC, a graph $G = (V, E)$
- Output an instance of TSP, a complete graph $G' = (V', E')$ and a target length T.

- where G' has a Hamiltonian Cycle of length $\leq T$ iff G has a Hamiltonian Cycle

- construct an instance of TSP with vertices V, and edge weights

$$d_{ij} = \begin{cases} 0 & \text{if } (i,j) \in E \\ 1 & \text{if } (i,j) \notin E \end{cases}$$

and a target length $T = 0$