Assignment 3

Due: February 4, 2020

Your solutions must be typed (preferably typeset in LaTeX) and submitted as a hard-copy at the beginning of class on the day its due.

Problem 1: Lower Bounds [10 points] Prove that any comparison-based algorithm for constructing a binary search tree from an arbitrary list of n elements takes $\Omega(n \log n)$ time in the worst case. (Hint: Consider how to reduce the sorting problem to performing a set of operations on a binary search tree. In other words, show that if a faster algorithm existed for constructing a binary search tree then you would violate the $\Omega(n \log n)$ comparison-based sorting lower bound.)

Problem 2: Median of Medians The 'Median-of-medians' selection algorithm presented in class divides the input into groups of 5. Using a group of odd size helps keep things a little simpler (because otherwise the group medians are messier to define), but why the choice of 5?

(a) [10 points] Show that the same argument for linear worst-case time complexity works if we use groups of size 7 instead.

(b) [10 points] Show that groups of size 3 results in superlinear time complexity.