Answer the questions in the spaces provided. If you run out of room for an answer, please continue on the back of the page.

Name: _____

Do you plan to collect your exam tonight: _____

Answer whether the following statements are true or false and briefly explain your answer in the space provided.

1. [TRUE / FALSE] If a problem is NP-complete and $P \neq NP$ then all instances of that problem require exponential time to solve. [5 pts]

2. [TRUE / FALSE] If $A$ is in NP and $B$ is in P, then $A \cap B$ cannot be NP-Complete unless $P = NP$. [5 pts]

3. [TRUE / FALSE] If $A$ is NP-Complete and $A \leq_p B$, then $B$ must be NP-Complete. [5 pts]

4. [TRUE / FALSE] For any language $L$, $L$ is polynomial-time reducible to some problem in $NP$ if and only if $L \in NP$. [5 pts]

5. [TRUE / FALSE] There exists a problem that is solvable in finite time, but is not solvable in Poly-time. [5 pts]

6. [TRUE / FALSE] If $P = NP$ then every language $L \in NP$ is NP-Complete except for $\emptyset$ and $\Sigma^*$. [5 pts]

7. For each of the following assertions, state whether they are $True$, $False$ or $Open$ according to our current state of knowledge of complexity theory as described in class. You DO NOT need to justify your answer choice.

   (a) $TIME(n) \subset TIME(n^2)$ [2 pts]

   (b) P $\subset$ (NP $\cap$ Co-NP) [2 pts]

   (c) P contains all regular languages [2 pts]

   (d) No undecidable language is an element of NP [2 pts]

   (e) NP-Complete $\subset$ NP [2 pts]

8. Consider the two decision problems below and answer the three question that follow (one is on the next page).

   *Problem 1:* Given an undirected graph $G = (V, E)$ with $|V|$ even, does $G$ contain a clique with at least $\frac{|V|}{2}$ vertices?

   *Problem 2:* Given an undirected graph $G = (V, E)$, does $G$ contain a clique with at least $|V| - 2$ vertices?

   (a) Which of the two problems is in P and which is NP-Complete? [3 pts]

   (b) For the problem in P, describe a poly time decider for the language. [7 pts]

(c) For the other problem, prove that it is NP-Complete. [10 pts]

9. Prove that Poly-time reductions are transitive. That is: if $A \leq_p B$ and $B \leq_p C$ then $A \leq_p C$. [10 pts]

10. Let $DoubleSat = \{\langle \Phi \rangle \mid \Phi \text{ has at least 2 satisfying assignments}\}$. Show that $DoubleSat$ is NP-Complete. [10 pts]

11. Prove that the following language is in NP. [10 pts]

$$\{\langle x, k\rangle \mid x \in \mathbb{N} \text{ and has a prime factor that is } < k$$

12. Prove that the following language is in Co-NP. [10 pts]

$$\{\langle x, k\rangle \mid x \in \mathbb{N} \text{ and has a prime factor that is } < k$$