Answer the questions in the spaces provided. If you run out of room for an answer, please continue on the back of the page.

Name: _____

1. Answer whether the following statements are true or false and briefly explain your answer.

   (a) [TRUE / FALSE] The class of Turing-Recognizable languages is closed under the subset operation. [5 pts]

   (b) [TRUE / FALSE] The set of all Turing-undecidable languages is countable. [5 pts]

(c) [TRUE / FALSE] If a language A is Turing-decidable and $A \leq_m B$ then B is Turing-decidable. [5 pts]

(d) [TRUE / FALSE] For any undecidable language, L, the complement of L is unrecognizable. [5 pts]

(e) [TRUE / FALSE] The class of Turing-Decidable languages are closed under the perfect-shuffle operation. [5 pts]

2. Let $INF_{DFA} = \{\langle D \rangle \mid D \text{ is a DFA and } L(D) \text{ is infinite}\}$. Show that INF_{DFA} is decidable. [15 pts]

3. Let $\Sigma = \{0, 1\}$. Show that the problem of determining whether a CFG generates at least one string in 1^* is decidable. This can be written as the following language: [15 pts]

$$\{\langle G \rangle \mid G \text{ is a CFG and } 1^* \cap L(G) \neq \emptyset\}$$

4. Prove that the class of Turing-Recognizable languages are closed under concatenation. [15 pts]

5. Prove that the following language is undecidable. [15 pts]

$$\{\langle M \rangle \mid M \text{ is a TM and that halts on the empty string}\}$$

6. Let $L_{TM} = \{\langle M \rangle \mid M \text{ is a TM that loops on all inputs}\}$. Prove that L_{TM} is not Turing-Recognizable. [15 pts]