Answer the questions in the spaces provided. If you run out of room for an answer, please continue on the back of the page.

Name: ______________________________

Answer whether the following statements are true or false and briefly explain your answer.

1. [TRUE / FALSE] The intersection of any context-free languages A and B is not context-free. [5 pts]

2. [TRUE / FALSE] There exists a DFA that recognizes all decimal numbers that are valid signed 64 bit integers. [5 pts]
   i.e. all numbers in the range -9,223,372,036,854,808 to 9,223,372,036,854,807

3. [TRUE / FALSE] Any language over some alphabet can be converted to an equivalent language over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

4. [TRUE / FALSE] If L is a context-free language and $A \subset L$ then A is context-free. [5 pts]

5. [TRUE / FALSE] For every regular language, L, where the minimal DFA for L has k states, there exists an NFA for L with fewer than k states. [5 pts]

6. [TRUE / FALSE] Given an NFA M where the $L(M) = A$, you can create an NFA for the complement of A by making every accept state in M a non-accepting state and every non-accepting state in M an accepting state. [5 pts]

7. [TRUE / FALSE] For any DFA D you can create a new DFA D' that accepts the same language and has a single accept state. [5 pts]

8. [TRUE / FALSE] For all Regular languages there exists an unambiguous Context Free Grammar. [5 pts]

9. For each language identify whether that language is Regular, Context-Free or Neither. If the language is Regular, give a DFA, NFA, or RegEx for the language. If the language is Context-Free and not Regular, give a CFG for the language and use the Pumping lemma or the Myhill-Nerode theorem to show that it isn't Regular. The the language is not Context-Free, use the Context-Free Pumping lemma.

   (a) $\{w \mid w \in \{0,1\}^*$ and w contains an equal number of substrings 01 and 10$\}$ [10 pts]

(b) $\{w \mid w \in \{0,1\}^*$ and w is a binary number of the form $2^k + 1$ for some $k \geq 0\}$ [10 pts]

(c) $\{0^n1^m0^m1^n \mid n, m \geq 0\}$ [10 pts]

(d) $\{w\overline{w} \mid w, \overline{w} \in \{0,1\}^*\}$ where $\overline{w}$ is w but where zeroes are replaced with ones and vice verse. Example: if $w = 1011$ then $\overline{w} = 0100$. [10 pts]

10. Let the *rotation closure* of a language L be $RC(L) = \{yx \mid xy \in L\}$. Prove or disprove that if L is a Regular language then the *rotational closure* of L is also Regular. [10 pts]

11. Prove or disprove that if L is a Context-Free language then the reverse, L^R, is also Context-Free. [10 pts]
L^R is defined as:

$$\{w^R \mid w \in L\}$$