Pushdown Automata (PDA) Formal Definition

A finite automaton that employs a stack.

A PDA is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$

Q is a set of states

Σ is the input alphabet

Γ is the stack alphabet

$\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow \mathcal{P}(Q \times \Gamma_\varepsilon)$

$q_0 \in Q$ is the start state

$F \subseteq Q$ is the set of accept states

The next move is determined by the

current state

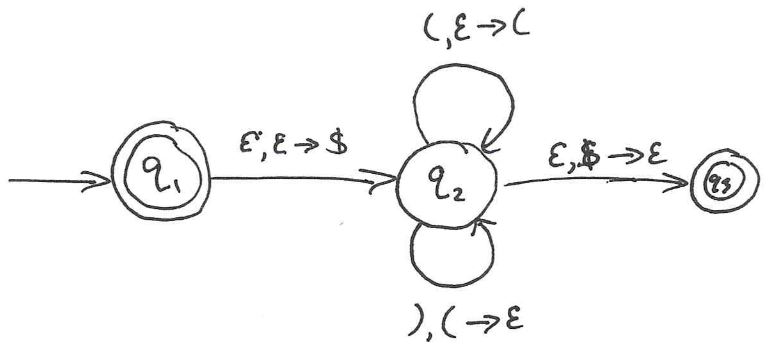
next input symbol read

top symbol of the stack.

Outputs

new state

new top symbol for the stack



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$\{ w \mid w$ is a set of balanced parenthesis $\}$

$$S \to (S) \mid SS \mid \varepsilon$$

A language is context-free iff some PDA recognizes it.

Drawing PDAs for anything more than simple context-free languages is very difficult.

Computation on a PDA

A PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ accepts $w = w_1 w_2 \dots w_n$ where each $w_i \in \Sigma_\varepsilon$ and there is a sequence of states $r_0 r_1 r_2 \dots r_n \in Q$ and strings $s_0 s_1 s_2 \dots s_n \in \Gamma^*$ exist that satisfy

1. $r_0 = q_0$ and $s_0 = \varepsilon$

   M starts in the start state with an empty stack.

2. For $i = 0 \dots n-1$

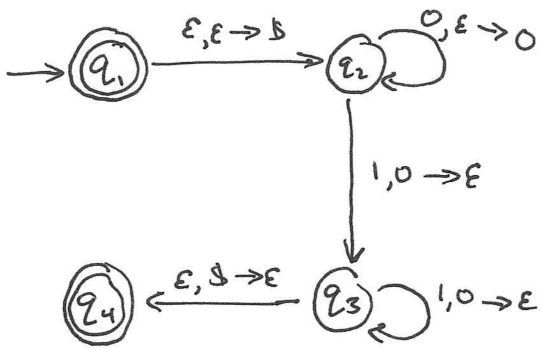
   $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$ where

   $s_i = at$ and $s_{i+1} = bt$

   for some $a, b \in \Gamma_\varepsilon$ and $t \in \Gamma^*$

3. $r_n \in F$

Example



$$\{0^n 1^n \mid n \geq 0\}$$

Theorem

If a language is context-free some PDA recognizes it.

Turn a CFG into a PDA

Allow an intermediate representation that is allowed to push multiple symbols on the stack at a time.

Starting with a CFG G create a PDA P

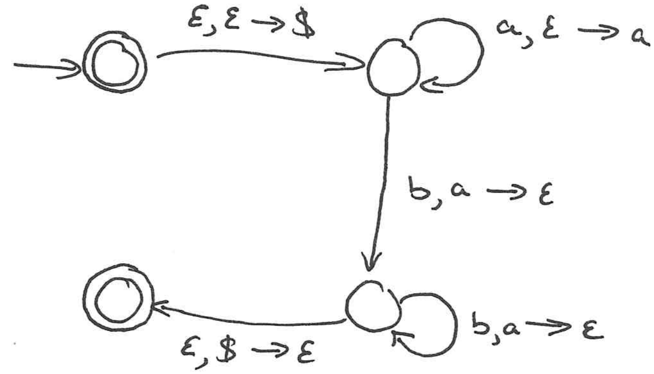
1. Place \$ and the start variable on the stack.
2. Repeat the following
   a.) If the top of the stack is a variable, A nondeterministically select one of the rules for A and substitute A with the string on the right-hand side of the rule.
   b.) If the top of the stack is a terminal a read the next symbol from the input and compare. If they match repeat. If they don't reject.
   c) If the top of the stack is \$ and the input is empty, enter the accept state.

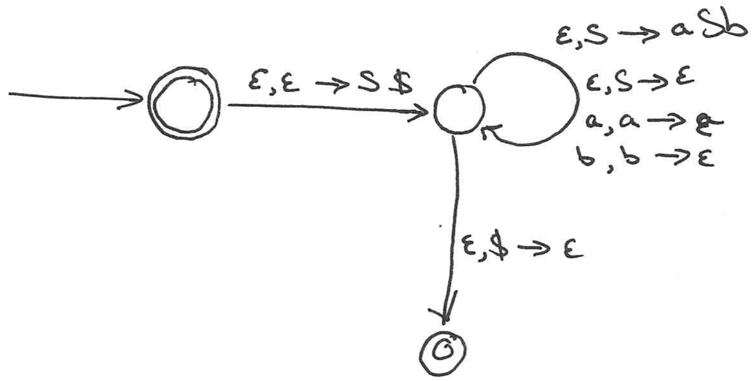
Example 1

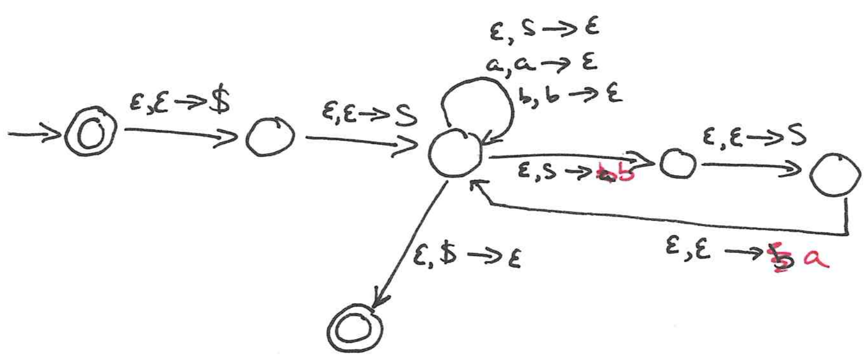
$\{a^n b^n \mid n \geq 0\}$

$S \rightarrow aSb$

$S \rightarrow \varepsilon$



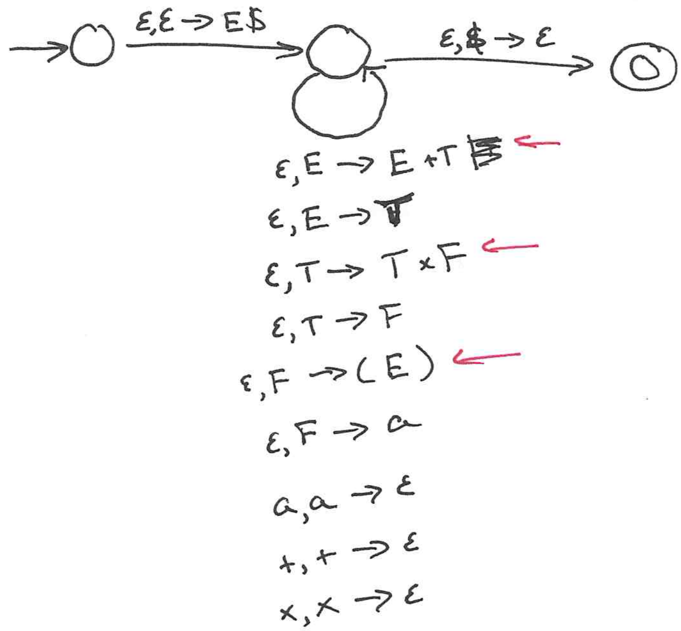


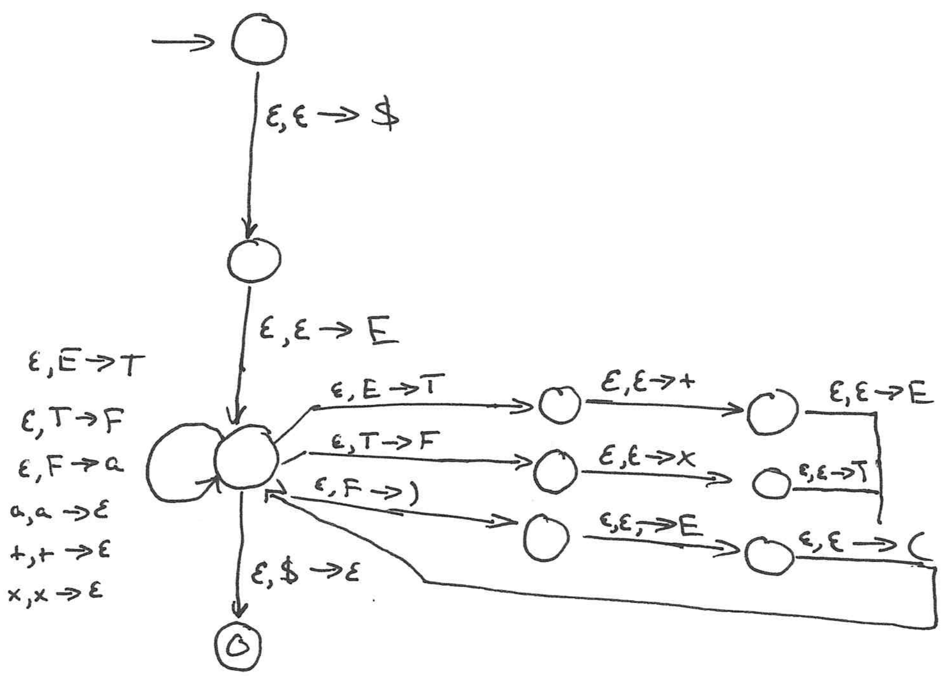


$E \to E + T \mid T$

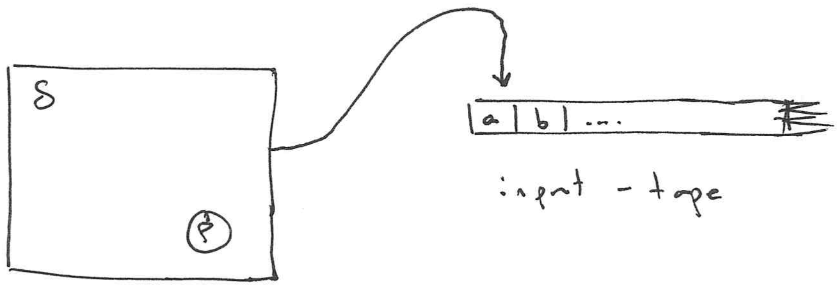
$T \to T \times F \mid F$

$F \to (E) \mid a$





DFA



Every time we transition states we read a symbol from the input tape.

PDA