Why is the intuitive description of a DFA called a state machine?

$A = \{ w \mid w \text{ ends in } 111 \}$

How many states does a machine that recognizes A need to have? 4

Define a DFA M such that $L(M) = A$

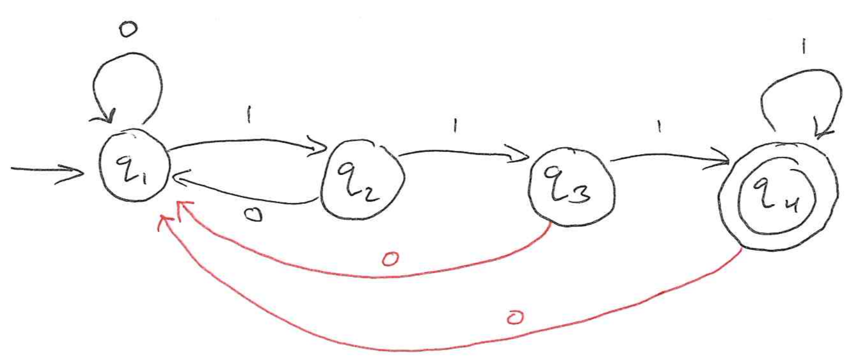
Possible states

1. 0 1s in a row

2. one 1 in a row

3. two 1s in a row

4. three or more 1s in a row



$A = \{ w \mid w \text{ is an even number represented in base } 3 \}$

base 3	base 10
0	0
1	1
2	2
10	3
11	4
12	5
20	6
21	7
22	8
100	9
101	10

Formal Definition of

Finite Automata 5-tuple

$$\boxed{(Q, \Sigma, \delta, q_0, F)}$$

Q = Finite set of states

Σ = finite set of symbols

$\delta : Q \times \Sigma \to Q$ transition function

$q_0 \in Q$ start state

$F \subseteq Q$ set of accept states

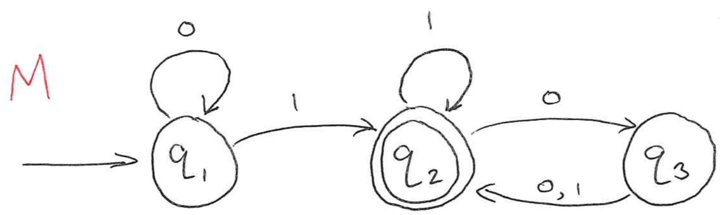
Does a DFA need a transition for every input from every state?

yes, otherwise δ wouldn't be a function

Can F be empty? yes

What language is recognized when F is empty? $\emptyset$

M



Formal definition of M?

$Q = \{q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$q_0 = q_1$

$F = \{q_2\}$

δ	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

How do DFAs relate to languages?

recognize

Can a DFA recognize more than one string?

yes

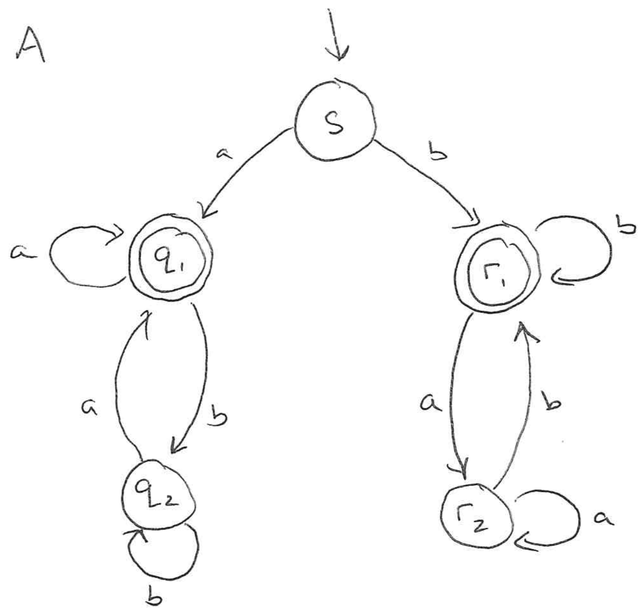
What language is recognized by M?

Can a DFA recognize more thane one language?

No

$L(M) = \{ w \mid w$ contains at least one 1 and an even number of 0s following the last 1$\}$

A



$Q = \{s, q_1, q_2, r_1, r_2\}$

$\Sigma = \{a, b\}$

$q_0 = s$

$F = \{q_1, q_2\}$

δ	a	b
s	q_1	r_1
q_1	q_1	q_2
q_2	q_1	q_2
r_1	r_2	r_1
r_2	r_2	r_1

What language is recognized by the DFA A?

$L(A) = \{ w \mid w$ starts and ends with the same letter$\}$

What changes would have to be made to create the language

$\{ w \mid w$ starts and ends with different letters$\}$

What is a regular language?

Def A language is called regular if some finite automaton recognizes it.

Is this definition helpful? not really

The collection of Regular Languages can be defined recursively.

- The empty language $\emptyset$ is a regular language
- $\forall a \in \Sigma$, the singleton language $\{a\}$ is a regular language.
- If A and B are regular languages then.
  - $A \cup B$ union
  - $A \circ B$ concatenation
  - A^* kleene star

  are regular languages
- No other language is regular.

Can we create the language already shown using only these rules?

yes

Closure:

what operations are the Natural numbers closed under?

$+ \quad \times$

and not closed under?

$\div \quad -$

Theorem

The class of regular languages is closed under union. If A and B are regular languages then $A \cup B$ is a regular language.

Since A and B are regular languages then there is a DFA M_a that recognizes A and a DFA M_B that recognizes B.

Proof by Construction

Create a DFA M that recognizes $A \cup B$

machine M must accept exactly when either M_A or M_B would accept in order to recognize $A \cup B$

Let M_A recognize A where $M_a = (Q_A, \Sigma_A, \delta_A, q_A, F_A)$

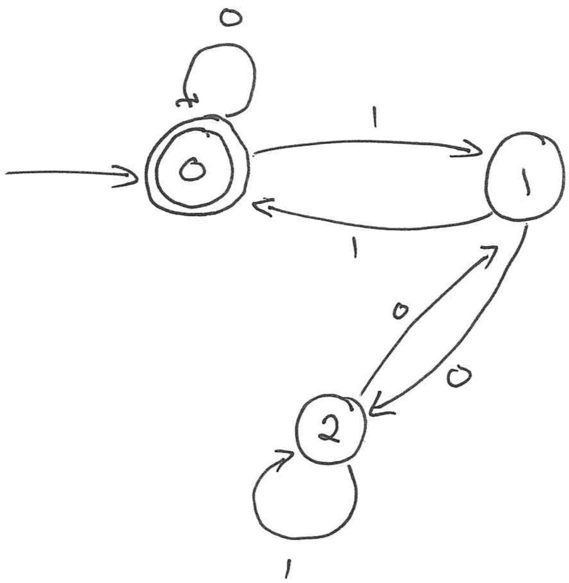
Let M_B recognize B where $M_B = (Q_B, \Sigma_B, \delta_B, q_B, F_B)$

Construct M to recognize $A \cup B$, where $M = (Q, \Sigma, \delta, q_0, F)$

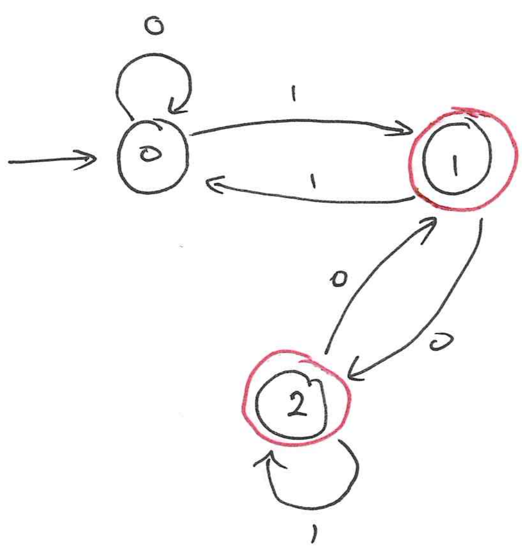
- $Q = \{(r_A, r_B) \mid r_A \in Q_A \text{ and } r_B \in Q_B\}$ i.e. Q is the cartesian product of Q_A and Q_B

  $Q_A \times Q_B$
- $\Sigma = \Sigma_A \cup \Sigma_B$
- $\delta((r_A, r_B), a) = (\delta(r_A, a), \delta(r_B, a))$
- $q_0 = (q_A, q_B)$
- $F = \{(r_A, r_B) \mid r_A \in F \text{ or } r_B \in F\}$ $(F_A \times Q_B) \cup (Q_A \times F_B)$

what is the result if $F_A \times F_B$?



1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001



Formal Definition of Computation

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton

let $w = w_1 w_2 \ldots w_n$ be a string where each

$w_i \in \Sigma$

M ACCEPTS w if a sequence of states

$r_0, r_1, \ldots r_n$ where each $r_i \in Q$

exists with the following conditions.

1. $r_0 = q_0$ must start in the start state
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for $i = 0, \ldots, n-1$ the machine must go from state to state according to the transition function
3. $r_n \in F$

machine accepts if it ends in an accept state