# Why Study Theory of Computation

Generally T.C lays a strong foundation for many abstract areas of CS.

## Automata Theory

- Reg languages are a powerful tool by themselves
  Reg Ex
- State machines even be used to represent many programs and systems
  - also relate to certain areas of math
  - computer games
- CFL directly apply to Programming language design
- Turing Machines represent everything a computer can do

## Computation Theory

- explores the limits of what can be done with a computer at all

Proof

Theorem: $\sqrt{2}$ is irrational

Proof by contradiction.

Assume $\sqrt{2}$ is rational. If $\sqrt{2} \in \mathbb{Q}$

$$\sqrt{2} = \frac{m}{n}$$

for some $m, n \in \mathbb{Z}$.

If $m, n$ are divisible by an integer greater than 1, divide both by the largest such integer.

Either $m$ or $n$ is odd.

$$\sqrt{2} = \frac{m}{n}$$

$$n\sqrt{2} = m$$

$$2n^2 = m^2$$

This shows us that $m$ is even since $n^2 \in \mathbb{Z}$ and an odd times an odd is always odd. If $m$ is even $m = 2k$ for some integer $k$. Substitute $2k$ for $m$

$$2n^2 = (2k)^2$$

$$2n^2 = 4k^2$$

$$n^2 = 2k^2$$

Therefore $n$ is also even. which introduces a contradiction since we reduced $m$ and $n$ so that they were not both even.

This term we will focus on Languages

a language is a set of string over an alphabet $L \subseteq \Sigma^*$ $\Sigma$ usually $\{0, 1\}$

And Decision Problems

given some input we answer yes/no 1/0

# Alphabet

Def: a finite non-empty set ($\Sigma$)

The elements of an alphabet are called letters or symbols

examples

$\{0, 1\}$ - binary alphabet

$\{\cdot, -\}$ - morse code

$\{a, b, c, \ldots, z\}$ - lower case letters

$\{\uparrow, \downarrow, \leftarrow, \rightarrow, a, b, \text{select}, \text{start}\}$

we will typically use the binary alphabet or some subset of the lowercase letters.

A string or word is a sequence of letter from the alphabet

The set of all strings that can be made from an alphabet $\Sigma$ is written $\Sigma^*$

called the Kleene star or Kleene Closure after Stephen Cole Kleene a student of Alonzo Church along with Alan Turing. Laid the foundation of Recursion Theory

# Language

Def: a set of strings over an alphabet

$$L \subseteq \Sigma^*$$