It is recommended that you read through the exam before you begin. Answer all questions in the space provided.

Name: _____

Answer whether the following statements are True or False and provide a brief proof sketch or counterexample to support your answer.

1. [TRUE / FALSE] Any algorithm with a pair of nested loops has a complexity of $\Omega(n^2)$. [5 pts]

2. [TRUE / FALSE] Because the Travelling Salesman problem is NP-Hard, no large instances, e.g. millions of cities, can be solved quickly. [5 pts]

3. [TRUE / FALSE] When using dynamic programming memoization will always perform better than tabulation since it avoids calculating any subproblem that isn't needed for the final solution. [5 pts]

4. [TRUE / FALSE] There exists a problem that we can prove can't be solved in poly-time. [5 pts]

5. (a) What is the best case space complexity of Breadth-first Search? (You do not need to prove this, just briefly describe your reasoning.) [5 pts]

(b) What is the worst case space complexity of Breadth-first Search? (You do not need to prove this, just briefly describe your reasoning.) [5 pts]

6. The Knapsack problem is NP-Hard, which means that a poly-time solution for it would prove that $P = NP$. Explain why the $O(nC)$ dynamic programming we showed in class, where n is the number of items and C is the capacity of our sack, does not prove that $P = NP$. [10 pts]

7. Where in a Max-Heap can the smallest element reside, assuming all elements are distinct? Include both the location in the array and the location in the implicit tree structure. [5 pts]

8. There is no known Greedy strategy that is optimal for solving the 0/1 Knapsack problem. For each of the following strategies give a counterexample, i.e. descibe an instance where that strategy will fail to produce an optimal result.

   (a) Lightest item first. [5 pts]

   (b) Most valuable item first. [5 pts]

   (c) Item with the best value to weight ratio first. [5 pts]