True/False Questions

1. Answer whether the following statements are true or false and briefly explain your answer.
   (a) [TRUE / FALSE] There exists a finite language that cannot be represented with a DFA. [5 pts]
   (b) [TRUE / FALSE] For any NFA N you can create a new NFA N' that accepts the same language and has a single accept state. [5 pts]
   (c) [TRUE / FALSE] For any DFA D you can create a new DFA D' that accepts the same language and has a single accept state. [5 pts]
   (d) [TRUE / FALSE] For any context-free language, there exists a PDA that decides that language. [5 pts]
   (e) [TRUE / FALSE] The intersection of a regular language and a context-free language is context-free. [5 pts]
   (f) [TRUE / FALSE] Any ambiguous CFG can be converted into an equivalent unambiguous CFG. [5 pts]

Language Identification

2. For each of the following languages, determine if it is regular, context-free, or neither. Briefly explain your reasoning.
   (a) $\{a^*b^*c^*\} - \{a^nb^nc^n \mid n \geq 0\}$ [5 pts]
      A. Regular Language
      B. Context-Free Language
      C. Neither
   (b) $\{w \mid w \in \{a, b\}$ contains an equal number of a's and b's$\}$ [5 pts]
      A. Regular Language
      B. Context-Free Language
      C. Neither
   (c) $\{0^k u 0^k \mid k \geq 1$ and $u \in \Sigma^*\}$ [5 pts]
      A. Regular Language
      B. Context-Free Language
      C. Neither

Drawing DFAs

3. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$$\{w \mid w \text{ contains the substring } 10001\}$$

4. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ has exactly 2 } a\text{'s and at least 2 } b\text{'s}\}$$

5. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ has an even number of a's and one or two b's}\}$$

6. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ starts with an a and has at most two b's}\}$$

7. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ has an even length and an odd number of a's}\}$$

8. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ is not an element of } a^*b^*\}$$

9. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ is any string that does not contain exactly 3 a's}\}$$

10. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$$\{w \mid w \text{ contains the substring 0101}\}$$

11. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$$\{w \mid w \text{ has length} \geq 3 \text{ and the third symbol is a 0}\}$$

12. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ has length of at most 5}\}$$

13. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \in a^n b^n \text{ where n} \leq 2\}$$

14. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$$\{w \mid \text{ every odd position in w contains a 1}\}$$

15. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{a, aa, aaa, b, bb, bbb\}$$

16. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$\{w \mid w$ contains an even number of a's and an odd number of b's and does not contain the substring ab$\}$

(hint: can you describe this language more simply)

17. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$\{w \mid w$ is all strings that do not contain a pair of 1's that are seperated by an odd number of symbols$\}$

18. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ has at least three } a\text{'s and at least 2 } b\text{'s}\}$$

19. Draw a DFA that recognizes the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ has an even number of a's and each a is followed by at least one b}\}$$

Drawing NFAs

20. Draw an NFA that recognizes the following language over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$$\{w \mid w \in 0^*\} \text{ with 1 state}$$

21. Draw an NFA that recognizes $A \circ B$ for the following languages A and B over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$$A = \{w \mid w \text{ contains an even number of 0's}\}$$

$$B = \{w \mid w \text{ ends with 0011}\}$$

22. Draw an NFA that recognizes $A \circ B$ for the following languages A and B over the alphabet $\Sigma = \{0, 1\}$. [5 pts]

$$A = \{w \mid w \text{ contains a 0 in every odd numbered space}\}$$

$$B = \{w \mid w \text{ ends with 000}\}$$

23. Draw an NFA that recognizes the following language over the alphabet $\Sigma = \{a, b, c, d\}$. [5 pts]

$$\{w \mid w \text{ does not contain every symbol in} \Sigma\}$$

Regular Language Pumping Lemma

24. Use the regular language pumping lemma to show that each of the following languages are not regular.
    (a) $\{0^n 1^n 2^n \mid n \geq 0\}$ [10 pts]
    (b) $\{w \in \{a, b, c\}^* \mid w \text{ contains more } a\text{'s than } b\text{'s}\}a$ [10 pts]
    (c) $\{a^{2^n} \mid n \geq 0\}$ [10 pts]
    (d) $\{ww \mid w \in \{a, b\}^*\}$ [10 pts]
    (e) $\{a^m b^n \mid n > m\}$ [10 pts]
    (f) $\{a^p \mid p \text{ is prime}\}$ [10 pts]
    (g) $\{w \mid w \in \{a, b\}^* \text{ and } w \text{ contains an equal number of } a\text{'s and } b\text{'s }\}$ [10 pts]

Regular Language Proofs

25. Prove that the language: [10 pts]

$$\{w \mid w \text{ is a multiple of } k \text{ represented in binary}\}$$

is regular for all finite values of k. (Hint: You need to describe a general construction for all k. Recall the homework and try some small values of k if you need to see the pattern.)

26. Prove that every NFA can be converted to an equivalent one with a single accept state. [5 pts]

27. For the following languages over tha alphabet, $\Sigma = \{0, 1\}$: [10 pts]

$$WW_k = \{ww \mid w \in \Sigma^* \text{ and } w \text{ is of length } k\}$$

   (a) Show that for each k, no DFA can recognize WW_k with fewer than 2^k states.

   (b) Describe a much smaller NFA for $\overline{WW_k}$, the complement of WW_k.

28. We define the *avoids* operation for languages A and B to be [10 pts]

$$A \textit{ avoids } B = \{w \mid w \in A \text{ and } w \text{ doesn't contain any string in } B \text{ as a substring}\}$$

   Prove that the class of regular languages is closed under the *avoids* operation. (Hint: you can construct the *avoids* operation using things we've already proven in class)

Context-Free Grammars

29. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ the length of } w \text{ is odd}\}$$

30. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{\text{ The empty set}\}$$

31. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ contains more } a\text{'s then } b\text{'s}\}$$

32. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{a^i b^j \mid i \neq j\}$$

33. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{x_1\#x_2\#...\#x_k \mid k \geq 1, \text{ each } x_i \in \Sigma^*, \text{ and for some } i \text{ and } j, x_i = x_j^R\}$$

34. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w\#x \mid w^R \text{ is a substring of } x\}$$

35. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ contains at least as many } a\text{'s as } b\text{'s}\}$$

36. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{xy \mid x, y \in \Sigma^* \text{ and } |x| = |y| \text{ but } x \neq y\}$$

37. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b\}$. [5 pts]

$$\{w \mid w \text{ does not contain the substring } aa\}$$

38. Write a context-free grammar for the following language over the alphabet $\Sigma = \{a, b, c\}$. [5 pts]

$$\{a^i b^j c^k \mid i = j \text{ or } j = k\}$$

Context-Free Pumping Lemma

39. Use the context-free pumping lemma to show that the following language over the alphabet, $\Sigma = \{a, b, c\}$ is not context-free. [10 pts]

$$B = \{a^i b^j c^k \mid i > j > k \geq 0\}$$

40. Use the context-free pumping lemma to show that the following language over the alphabet, $\Sigma = \{0, 1\}$ is not context-free. [10 pts]

$$C = \{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$$

41. Use the context-free pumping lemma to show that the following language over the alphabet, $\Sigma = \{a, b\}$ is not context-free. [10 pts]

$$\{www \mid w \in \{a, b\}^*\}$$

Context-Free Language Proofs

42. If A and B are regular languages then the following language C is context-free: [10 pts]

$$C = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$$