Problem 1  A Turing machine with left reset is similar to an ordinary Turing machine, but the transition function has the form

\[ \delta : Q \times \Gamma \to Q \times \Gamma \times \{ R, \text{RESET} \} \]

If \( \delta(q,a) = (r,b,\text{RESET}) \), when the machine is in state \( q \) reading an \( a \), the machine’s head jumps to the left-hand end of the tape after it writes \( b \) on the tape and enters state \( r \). Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages. [10 points]

(Hint: Much like previous problems in this class, you’ll need to describe some general construction that takes a Turing Machine-with-reset to an ordinary Turing Machine and visa-versa)

Problem 2  Give the informal descriptions for Turing machines that decide the following languages

a) \( \{ w \mid w \text{ contains twice as many 0s as 1s} \} \) [5 points]

b) \( \{ a + b = c \mid a, b, c \in \{0, 1\}^* \text{ and the binary numbers represented by } a \text{ and } b \text{ sum to } c \} \) [5 points]

Problem 3  Show that the Turing-decidable languages are closed under

a) union [5 points]

b) intersection [5 points]

c) complement [5 points]

d) set difference [5 points]

Problem 4  Show that the Turing-recognizable languages are closed under concatenation

This problem requires providing constructions that take individual Turing machines and combines them into a new machine that recognizes the new language. Remember, this is about Turing-recognizable languages not just decideable so that there’s a possibility of non-termination. [10 points]

Problem 5  For each of the following Turing machine variants determine if the machine is more powerful, equivalent, or less powerful than a single-tape Turing machine. If less powerful describe the class of languages recognized by the machine. Explain your answers.

a) A Turing Machine that can only make moves to the right and never left. [5 points]

b) A Turing Machine that can move right one space or move left two spaces. [5 points]

c) A Turing Machine that never writes to a space on the tape that already contains a symbol. [5 points]