## Assignment 3

CS 311, Fall 2015 Due: October 28, 2014

**Problem 1** For the language:

 $A = \{ w \in \{0,1\}^* \mid w \text{ is not a palindrome} \}$ 

- a) Using the pumping lemma, show that A is *not* regular. (Hint: the regular languages are closed under complement) [5 Points]
- b) Give a CFG that generates the language A. [5 Points]

**Problem 2** For the language:

 $B = \{w \in \{a, b\}^* \mid \text{The first, middle, and last character are the same}\}$ 

- a) Using the pumping lemma, show that B is not regular. [5 points]
- b) Give a CFG that generates the language B. [5 points]

**Problem 3** The languages  $\{0^{s}1^{s}2^{t} | s, t \ge 0\}$  and  $\{0^{s}1^{t}2^{t} | s, t \ge 0\}$  are context-free. Use those languages to answer the following:

- a) Write down CFGs for each of the languages. [5 Points]
- b) Use the fact that both languages are context-free to show that the class of context-free languages is *not* closed under intersection. (Use the context-free pumping lemma to show that the resulting language is not context-free.) [5 Points]
- c) Use the result from part b and DeMorgan's Laws to show that the class of context-free languages are *not* closed under complement. [5 Points]

**Problem 4** Give a Context-Free Grammar for the following language:

$$\{a^*b^*c^*\} - \{a^nb^nc^n \mid n \ge 0\}$$

(Hint: See if you can rephrase the language to make it easier)

**Problem 5** Write the context free grammar for  $\{w \mid w \text{ is a palindrome }\}$  and then convert the grammar into Chomsky normal form using the procedure from class. [10 Points]

**Problem 6** Prove or disprove: If G is a CFG in Chomsky normal form, then for any string  $w \in L(G)$  of length  $n \ge 1$  then exactly 2n - 1 steps are required for any derivation of w. [10 Points]