# Assignment 2

CS 311, Fall 2015
Due: October 14, 2014

**Problem 1** Give state diagrams of DFAs recognizing the following languages.

a) $\{w \mid w \text{ contains the substrings } ab \text{ and } ba\}$, $\Sigma = \{a, b\}$ [5 points]

b) $\{w \mid w \text{ contains an even number of 0s or exactly three 1s}\}$, $\Sigma = \{0, 1\}$ [5 points]

c) $\{w \mid w \text{ is a binary multiple of 5}\}$, $\Sigma = \{0, 1\}$ [5 points]

d) $\{w \mid w = a^n b^n, 0 \leq n \leq 3\}$, $\Sigma = \{a, b, c\}$ [5 points]

**Problem 2** Prove or disprove the following: Let $D$ be a DFA with $|Q| = k$. If $|L(D)|$ is finite then there exists a string $w$ of length at most $k - 1$ such that $w \notin L(D)$.[10 points]

**Problem 3** For any string $w = w_1 w_2 ... w_n$, the reverse of $w$, written $w^R$, is the string $w$ in reverse order, $w_n ... w_2 w_1$. For any language $A$, let $A^R = \{w^R \mid w \in A\}$. Show that if $A$ is regular, so is $A^R$. [10 points]

**Problem 4** Prove that the language:

$$\{w \mid w \text{ is a multiple of } k \text{ represented in binary}\}$$

is regular for all finite values of $k$. (Hint: You need to describe a general construction for all $k$. Consider how your DFA from problem 1c relates to bit-shifting) [10 points]

**Problem 5** Describe a language $L$ for which there exists an NFA $N$ that has a small number of states (think linear on some property of the language), but a DFA $M$ must have a large number of states (think exponential on some property of the language). Clearly and carefully justify your answer. [10 points]