Problem 1  Prove by induction on $n$:

$$\forall a \geq 2, \sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$$

Problem 2  Use the pigeonhole principle to prove the following: Every undirected simple graph with at least 2 nodes has two nodes with the same degree.

Problem 3  Let $S = \{a, b, c, d, e, f\}$, find the number of subsets of $S$ which include the elements $a$ or $b$.

Problem 4  Given two natural numbers $x$ and $y$ such that $x > y$, prove that:

$$x \mod y < \frac{x}{2}$$