### Chapter 3  Measurement System Behavior

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y^0(0))</td>
<td>initial condition of (y^0)</td>
</tr>
<tr>
<td>(A)</td>
<td>input signal amplitude</td>
</tr>
<tr>
<td>(B(\omega))</td>
<td>output signal amplitude</td>
</tr>
<tr>
<td>(C)</td>
<td>constant</td>
</tr>
<tr>
<td>(E(t))</td>
<td>voltage (V) or energy</td>
</tr>
<tr>
<td>(F)</td>
<td>force ((m l t^{-2}))</td>
</tr>
<tr>
<td>(F(t))</td>
<td>forcing function</td>
</tr>
<tr>
<td>(G(s))</td>
<td>transfer function</td>
</tr>
<tr>
<td>(K)</td>
<td>static sensitivity</td>
</tr>
<tr>
<td>(M(\omega))</td>
<td>magnitude ratio, (B/KA)</td>
</tr>
<tr>
<td>(T(t))</td>
<td>temperature (\left({}^\circ\right))</td>
</tr>
<tr>
<td>(T_d)</td>
<td>ringing period ((t))</td>
</tr>
<tr>
<td>(U(t))</td>
<td>unit step function</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>time lag ((t))</td>
</tr>
<tr>
<td>(\delta(\omega))</td>
<td>dynamic error</td>
</tr>
<tr>
<td>(\tau)</td>
<td>time constant ((t))</td>
</tr>
<tr>
<td>(\Phi(\omega))</td>
<td>phase shift</td>
</tr>
<tr>
<td>(\omega)</td>
<td>circular frequency ((t^{-1}))</td>
</tr>
<tr>
<td>(\omega_n)</td>
<td>natural frequency magnitude ((t^{-1}))</td>
</tr>
<tr>
<td>(\omega_d)</td>
<td>ringing frequency ((t^{-2}))</td>
</tr>
<tr>
<td>(\omega_R)</td>
<td>resonance frequency ((t^{-1}))</td>
</tr>
<tr>
<td>(\xi)</td>
<td>damping ratio</td>
</tr>
<tr>
<td>(\Gamma)</td>
<td>error fraction</td>
</tr>
</tbody>
</table>

**Subscripts**

- 0: initial value
- \(\infty\): final or steady value
- \(h\): homogeneous solution

### PROBLEMS

*Note: Although not required, the companion software can be used for solving many of these problems. We encourage the reader to explore the software provided.*

**3.1** A mass measurement system has a static sensitivity of 2 V/kg. An input range of 1 to 10 kg needs to be measured. A voltmeter is used to display the measurement. What range of voltmeter is needed. What would be the significance of changing the static sensitivity?

**3.2** Determine the 75%, 90%, and 95% response time for each of the systems given (assume zero initial conditions):

a. \(0.4\ddot{y} + \tau \dot{y} = 4U(t)\)

b. \(\ddot{y} + 2\dot{y} + 4y = U(t)\)

c. \(2\ddot{P} + 8\dot{P} + 8P = 2U(t)\)

d. \(5\ddot{y} + 5y = U(t)\)

**3.3** A special sensor is designed to sense the percent vapor present in a liquid–vapor mixture. If during a static calibration the sensor indicates 80 units when in contact with 100% liquid, 0 units with 100% vapor, and 40 units with a 50:50% mixture, determine the static sensitivity of the sensor.

**3.4** A measurement system can be modeled by the equation

\[0.5\ddot{y} + y = F(t)\]

Initially, the output signal is steady at 75 volts. The input signal is then suddenly increased to 100 volts.

a. Determine the response equation.

b. On the same graph, plot both the input signal and the system time response from \(t = 0\) s through steady response.
3.5 Suppose a thermometer similar to that in Example 3.3 is known to have a time constant of 30 s in a particular application. Plot its time response to a step change from 32° to 120°F. Determine its 90% rise time.

3.6 Referring back to Example 3.3, a student establishes the time constant of a temperature sensor by first holding it immersed in hot water and then suddenly removing it and holding it immersed in cold water. Several other students perform the same test with similar sensors. Overall, their results are inconsistent, with estimated time constants differing by as much as a factor of 1.2. Offer suggestions as to why this might happen. Hint: Try this yourself and think about control of test conditions.

3.7 A thermocouple, which responds as a first-order instrument, has a time constant of 20 ms. Determine its 90% rise time.

3.8 During a step function calibration, a first-order instrument is exposed to a step change of 100 units. If after 1.2 s the instrument indicates 80 units, estimate the instrument time constant. Estimate the error in the indicated value after 1.5 s. $y(0) = 0$ units; $K = 1$ unit/unit.

3.9 Estimate any dynamic error that could result from measuring a 2-Hz periodic waveform using a first-order system having a time constant of 0.7 s.

3.10 A signal expected to be of the form $F(t) = 10 \sin 15.7t$ is to be measured with a first-order instrument having a time constant of 50 ms. Write the expected indicated steady response output signal. Is this instrument a good choice for this measurement? What is the expected time lag between input and output signal? Plot the output amplitude spectrum; $y(0) = 0$ and $K = 1 \text{V/V}$.

3.11 A first-order instrument with a time constant of 2 s is to be used to measure a periodic input. If a dynamic error of ±2% can be tolerated, determine the maximum frequency of periodic input that can be measured. What is the associated time lag (in seconds) at this frequency?

3.12 Determine the frequency response [$M(\omega)$ and $\phi(\omega)$] for an instrument having a time constant of 10 ms. Estimate the instrument’s usable frequency range to keep its dynamic error within 10%.

3.13 A temperature measuring device with a time constant of 0.15 s outputs a voltage that is linearly proportional to temperature. The device is used to measure an input signal of the form $T(t) = 115 + 12 \sin 2t \degree$ C. Plot the input signal and the predicted output signal with time assuming first-order behavior and a static sensitivity of 5 mV/°C. Determine the dynamic error and time lag in the steady response. $T(0) = 115\degree$ C.

3.14 A first-order sensor is to be installed into a reactor vessel to monitor temperature. If a sudden rise in temperature greater than 100° C should occur, shutdown of the reactor will need to begin within 5 s after reaching 100°C. Determine the maximum allowable time constant for the sensor.

3.15 A single-loop LR circuit having a resistance of 1 MΩ is to be used as a low-pass filter between an input signal and a voltage measurement device. To attenuate undesirable frequencies above 1000 Hz by at least 50%, select a suitable inductor size. The time constant for this circuit is given by $L/R$.

3.16 A measuring system has a natural frequency of 0.5 rad/s, a damping ratio of 0.5, and a static sensitivity of 0.5 mV. Estimate its 90% rise time and settling time if $F(t) = 2 U(t)$ and the initial condition is zero. Plot the response $y(t)$ and indicate its transient and steady responses.

3.17 Plot the frequency response, based on Equations 3.20 and 3.22, for an instrument having a damping ratio of 0.6. Determine the frequency range over which the dynamic error remains within 5%. Repeat for a damping ratio of 0.9 and 2.0.

3.18 The output from a temperature system indicates a steady, time-varying signal having an amplitude that varies between 30° and 40°C with a single frequency of 10 Hz. Express the output signal as a
3.39 A signal suspected to be of the nominal form

\[ y(t) = 5 \sin(1000t) \text{ mV} \]

is input to a first-order instrument having a time constant of 100 ms and \( K = 1 \text{ V/V} \). It is then to be passed through a second-order amplifier having a \( K = 100 \text{ V/V} \), a natural frequency of 15,000 Hz, and a damping ratio of 0.8. What is the expected form of the output signal, \( y(t) \)? Estimate the dynamic error and phase lag in the output. Is this system a good choice here? If not, do you have any suggestions?

3.40 A typical modern DC audio amplifier has a frequency bandwidth of 0 to 20,000 Hz \( \pm 1 \text{ dB} \). Explain the meaning of this specification and its relation to music reproduction.

3.41 The displacement of a rail vehicle chassis as it rolls down a track is measured using a transducer \( (K = 10 \text{ mV/mm}, \omega_n = 10,000 \text{ rad/s}, \zeta = 0.6) \) and a recorder \( (K = 1 \text{ mm/mV}, \omega_n = 700 \text{ rad/s}, \zeta = 0.7) \). The resulting amplitude spectrum of the output signal consists of a spike of 90 mm at 2 Hz and a spike of 50 mm at 40 Hz. Are the measurement system specifications suitable for the displacement signal? (If not, suggest changes.) Estimate the actual displacement of the chassis. State any assumptions.

3.42 The amplitude spectrum of the time-varying displacement signal from a vibrating U-shaped tube is expected to have a spike at 85, 147, 220, and 452 Hz. Each spike is related to an important vibrational mode of the tube. Displacement transducers available for this application have a range of natural frequencies from 500 to 1000 Hz, with a fixed damping ratio of about 0.5. The transducer output is to be monitored on a spectrum analyzer that has a frequency bandwidth extending from 0.1 Hz to 250 kHz. Within the stated range of availability, select a suitable transducer for this measurement from the given range.

3.43 A sensor mounted to a cantilever beam indicates beam motion with time. When the beam is deflected and released (step test), the sensor signal indicates that the beam oscillates as an underdamped second-order system with a ringing frequency of 10 rad/s. The maximum displacement amplitudes are measured at three different times corresponding to the 1st, 16th, and 32nd cycles and found to be 17, 9, and 5 mV, respectively. Estimate the damping ratio and natural frequency of the beam based on this measured signal, \( K = 1 \text{ mm/mV} \).

3.44 Write a short essay on how system properties of static sensitivity, natural frequency, and damping ratio affect the output information from a measurement system. Be sure to discuss the relative importance of the transient and steady aspects of the resulting signal.

3.45 Burgess (5) reports that the damping ratio can be approximated from the system response to an impulse test by counting the number of cycles, \( n \), required for the ringing amplitudes to fall to within 1% of steady state by \( \zeta = (4.6/2n) \). The estimate is further improved if \( n \) is a noninteger. Investigate the quality of this estimate for a second-order system subjected instead to a step function input. Discuss your findings.

3.46 The starting transient of a DC motor can be modeled as an \( RL \) circuit, with the resistor and inductor in series (Figure 3.25). Let \( R = 4 \Omega, L = 0.1 \text{ H} \), and \( E_i = 50 \text{ V} \) with \( i(0) = 0 \). Find the current draw with time for \( t > 0^+ \).

3.47 A camera flash light is driven by the energy stored in a capacitor. Suppose the flash operates off a 6-V battery. Determine the time required for the capacitor stored voltage to reach 90% of its maximum energy \( (\frac{1}{2} CE_B^2) \). Model this as an \( RC \) circuit (Fig. 3.26). For the flash: \( C = 1000 \mu F, R = 1 \text{ k}\Omega \), and \( E_c(0) = 0 \).

**NOMENCLATURE**

- $a_0, a_1, \ldots, a_m$: polynomial regression coefficients
- $f_j$: frequency of occurrence of a value
- $x$: measured variable; measurement
- $x_i$: $i$th measured value in a data set
- $\bar{x}$: true mean value of the population of $x$
- $\langle x \rangle$: pooled mean value of $x$
- $p(x)$: probability density function of $x$
- $\bar{s}_x$: sample standard deviation of $x$
- $s_x$: sample standard deviation of the means of $x$; standard random uncertainty in $\bar{x}$
- $\langle s_x \rangle$: pooled sample standard deviation of $x$
- $s_y$: pooled standard deviation of the means of $x$
- $\langle s_y \rangle$: pooled sample variance of $x$
- $\langle s_y \rangle^2$: pooled sample variance of $x$
- $s_{yx}$: standard error of the (curve) fit between $y$ and $x$; standard random uncertainty in a curve fit
- $\bar{t}$: Student's $t$ variable
- $\beta$: normalized standard variate
- $\sigma$: true standard deviation of the population of $x$
- $\chi^2$: true variance of the population of $x$
- $\nu$: chi-squared value
- $\nu$: degrees of freedom

**PROBLEMS**

4.1 Determine the range of values containing 50% of the population of $x$. From a large sampling ($N > 5000$), we find that $x$ has a mean value of 5.2 units and a standard deviation of 1.0 units. Assume $x$ is normally distributed.

4.2 Determine the range of values containing 90% of the population of $x$. From a very large data set ($N > 50,000$), $x$ is found to have a mean value of 192.0 units with a standard deviation of 10 units. Assume $x$ is normally distributed.

4.3 At a fixed operating setting, the pressure in a line downstream of a reciprocating compressor has a mean value of 12.0 bar with a standard deviation of 1.0 bar based on a very large data set obtained from continuous monitoring. What is the probability that the line pressure will exceed 13.0 bar during any measurement?
4.4 Consider the toss of four coins. There are $2^4$ possible outcomes of a single toss. Develop a histogram of the number of heads (one side of the coin) that can appear on any toss. Does it look like a normal distribution? Should this be expected? What is the probability that three heads will appear on any toss?

4.5 As a game, slide a matchbook across a table, trying to make it stop at some predefined point on each attempt. Measure the distance from the starting point to the stopping point. Repeat this 10, 20, through 50 times. Plot the frequency distribution from each set. Would you expect them to look like a gaussian distribution? What statistical outcomes would distinguish a better player?

*Problems 4.6 through 4.15 refer to the three measured data sets in Table 4.8. Assume that the data have been recorded from three replications from the same process under the same fixed operating condition.*

4.6 Develop a histogram for the data listed in column 1. Discuss each axis and describe the overall shape of the histogram.

4.7 Develop a frequency distribution for the data given in column 3. Discuss each axis and describe its overall shape.

4.8 Develop and compare the histograms of the three data sets represented under columns 1, 2, and 3. If these are taken from the same process, why might the histograms vary? Do they appear to show a central tendency?

### Table 4.8 Measured Force Data for Exercise Problems

<table>
<thead>
<tr>
<th>Force (N) Set 1</th>
<th>Force (N) Set 2</th>
<th>Force (N) Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.9</td>
<td>51.9</td>
<td>51.1</td>
</tr>
<tr>
<td>51.0</td>
<td>48.7</td>
<td>50.1</td>
</tr>
<tr>
<td>50.3</td>
<td>51.1</td>
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<td>51.7</td>
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<td>49.7</td>
</tr>
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<tr>
<td>52.0</td>
<td>50.3</td>
<td>50.8</td>
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<tr>
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<td>50.2</td>
<td>50.8</td>
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<td>48.9</td>
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<tr>
<td>49.9</td>
<td>50.5</td>
<td>50.4</td>
</tr>
<tr>
<td>49.2</td>
<td>49.7</td>
<td>51.5</td>
</tr>
</tbody>
</table>
4.9 For the data in each column, determine the sample mean value, standard deviation, and standard deviation of the means. State the degrees of freedom in each.

4.10 Explain the concept of "central tendency" by comparing the range of the measured values and the sample mean values from each of the three data sets.

4.11 From the data in column 1, estimate the range of values for which you would expect 95% of all possible measured values for this operating condition to fall. Repeat for columns 2 and 3. Discuss these outcomes in terms of what you might expect from finite statistics.

4.12 From the data in column 1, determine the best estimate of the mean value at a 95% probability level. How does this estimate differ from the estimates made in problem 4.11? Repeat for columns 2 and 3. Why do the estimates vary for each data set? Discuss these outcomes in terms of what you might expect from finite statistics if these are measuring the same measured variable during the same process.

4.13 For the data in column 3, if one additional measurement were made, estimate the interval in which the value of this measurement would fall with a 95% probability.

4.14 Compute a pooled sample mean value for the process. State the range for the best estimate in force at 95% probability based on these data sets. Discuss whether this pooled sample mean value is reasonable given the sample mean values for the individual data sets. Write a short essay explanation in terms of the limitations of sample statistics, the number of measurements, variations in data sets, and statistical estimators.

4.15 Apply the $\chi^2$ goodness-of-fit test to the data in column 1 and test the assumption of a normal distribution.

4.16 Consider a process in which the applied measured load has a known true mean of 100 N with variance of 400 N$^2$. An engineer takes 16 measurements at random. What is the probability that this sample will have a mean value between 90 and 110?

4.17 A professor grades students on a normal curve. For any grade $x$, based on a course mean and standard deviation developed over years of testing, the following applies:

- A: $x > \bar{x} + 1.6\sigma$
- B: $\bar{x} - 0.4\sigma < x \leq \bar{x} + 1.6\sigma$
- C: $\bar{x} - 0.4\sigma < x \leq \bar{x} + 0.4\sigma$
- D: $\bar{x} - 1.6\sigma < x \leq \bar{x} - 0.4\sigma$
- F: $x \leq \bar{x} - 1.6\sigma$

How many A, C, and D grades are given per 100 students?

4.18 The production of a certain polymer fiber follows a normal distribution with a true mean diameter of 20 $\mu$m and a standard deviation of 30 $\mu$m. Compute the probability of a measured value greater than 80 $\mu$m. Compute the probability of a measured value between 50 and 80 $\mu$m.

4.19 An automotive manufacturer removes the friction linings from the clutch plates of drag race cars following test runs. A sampling of 10 linings for wear show the following values (in $\mu$m): 204.5, 231.1, 157.5, 190.5, 261.6, 127.0, 216.6, 172.7, 243.8, and 291.0. Estimate the average wear and its variance. Based on this sample, how many clutch plates out of a large set will be expected to show wear of more than 203 $\mu$m?

4.20 Determine the mean value of the life of an electric light bulb if

$$p(x) = 0.001e^{-0.001x}, x \geq 0$$

and $p(x) = 0$ otherwise. Here $x$ is the life in hours.
4.21 Compare the reduction in the possible range of random error in estimating \( x' \) by taking a sample of 16 measurements as opposed to only four measurements. Then compare 100 measurements to 25. Explain "diminishing returns" as it applies to using larger sample sizes to reduce random error in estimating the true mean value.

4.22 The variance in the strength test values of 270 bricks is 6.89 (MN/m\(^2\))^2 with a mean of 6.92 MN/m\(^2\). What is the random error in the mean value and the confidence interval at 95%?

4.23 During the course of a motor test, the motor rpm (revolutions per minute) is measured and recorded at regular intervals as:

990 1030 950 1050 1000 980

Calculate the mean value, standard deviation and the best estimate of the true value for this data set. Over what interval would 90% of the entire population of motor speed values fall? Test the data set for potential outliers.

4.24 A batch of rivets is tested for shear strength. A sample of 31 rivets shows a mean strength of 924.2 MPa with a standard deviation of 18 MPa. Estimate of the mean shear strength for the batch at 95% probability.

4.25 Three independent sets of data are collected from the population of a variable during similar process operating condition. The statistics are found to be

\[N_1 = 16; \bar{x}_1 = 32; s_{x_1} = 3 \text{ units}\]
\[N_2 = 21; \bar{x}_2 = 30; s_{x_2} = 2 \text{ units}\]
\[N_3 = 9; \bar{x}_3 = 34; s_{x_3} = 6 \text{ units}\]

Neglecting systematic errors and random errors other than the variation in the measured data set, compute an estimate of the pooled mean value of this variable and the range in which the true mean should lie with 95% confidence.

4.26 Eleven core samples of fresh concrete are taken by a county engineer from the loads of 11 concrete trucks used in pouring a structural footing. After curing, the engineer tests to find a mean compression strength of 3027 lb/in.\(^2\) with a standard deviation of 53 lb/in.\(^2\). State codes require a minimum strength of 3000 lb/in.\(^2\) at 95% confidence. Should the footing be repoured based on a 95% confidence interval of the test data?

4.27 The following data were collected by measuring the force load acting on a small area of a beam under repeated "fixed" conditions:

<table>
<thead>
<tr>
<th>Reading</th>
<th>Output (N)</th>
<th>Reading Number</th>
<th>Output (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>923</td>
<td>6</td>
<td>916</td>
</tr>
<tr>
<td>2</td>
<td>932</td>
<td>7</td>
<td>927</td>
</tr>
<tr>
<td>3</td>
<td>908</td>
<td>8</td>
<td>931</td>
</tr>
<tr>
<td>4</td>
<td>932</td>
<td>9</td>
<td>926</td>
</tr>
<tr>
<td>5</td>
<td>919</td>
<td>10</td>
<td>923</td>
</tr>
</tbody>
</table>

Determine if any of these data points should be considered outliers. If so, reject the data point. Estimate the true mean value from this data set assuming that the only error is from variations in the data set.
Chapter 5  Uncertainty Analysis

\[ \bar{x} \] sample mean value of \( x \)  
\[ R \] result or resultant value  
\[ P \] probability  
\[ B \] systematic uncertainty (at 95\% or 2\% probability)  
\( (P\%) \) confidence level  
\[ Q \] flow rate \((P^{-1})\)  
\[ T \] temperature \(^{\circ}\)  
\[ \forall \] volume \((L^{-3})\)  
\[ \theta_{i} \] sensitivity index  
\[ \rho \] gas density \((mL^{-3})\)  
\[ \sigma \] stress \((mL^{-1}T^{-2})\); population standard deviation  
\[ v \] degrees of freedom  
\[ ( ) \] pooled statistic

PROBLEMS

5.1 In Chapter 1, the development of a test plan is discussed. Discuss how a test plan should account for the presence of systematic and random errors. Include calibration, randomization, and repetition in your discussion.

5.2 Discuss how systematic uncertainty can be estimated for a measured value. How is random uncertainty estimated? What is the difference between error and uncertainty?

5.3 Explain what is meant by the terms “true value,” “best estimate,” “mean value,” “uncertainty,” and “confidence interval.”

5.4 Consider a common tire pressure gauge. How would you estimate the uncertainty in a measured pressure at the design stage and then at the Nth order? Should the estimates differ? Explain.

5.5 A micrometer has graduations inscribed at 0.001-in. (0.025-mm) intervals. Estimate the uncertainty due to resolution. Compare the results assuming a normal distribution to results assuming a rectangular distribution.

5.6 A tachometer has an analog display dial graduated in 5-revolutions-per-minute (rpm) increments. The user manual states an accuracy of 1\% of reading. Estimate the design-stage uncertainty in the reading at 50, 500, and 5000 rpm.

5.7 An automobile speedometer is graduated in 5-mph (8-kph) increments and has an accuracy rated to be within \pm 4\%. Estimate the uncertainty in indicated speed at 60 mph (90 kph).

5.8 A temperature measurement system is composed of a sensor and a readout device. The readout device has a claimed accuracy of 0.8\(^{\circ}\)C with a resolution of 0.1\(^{\circ}\)C. The sensor has an off-the-shelf accuracy of 0.5\(^{\circ}\)C. Estimate a design-stage uncertainty in the temperature indicated by this combination.

5.9 Two resistors are to be combined to form an equivalent resistance of 1000 \( \Omega \). The resistors are taken from available stock on hand as acquired over the years. Readily available are two common resistors rated at 500 \pm 50 \( \Omega \) and two common resistors rated at 2000 \( \Omega \) \pm 5\%. What combination of resistors (series or parallel) would provide the smaller uncertainty in an equivalent 1000 \( \Omega \) resistance?

5.10 An equipment catalog boasts that a pressure transducer system comes in 3\frac{1}{2}-digit (e.g., 19.99) or 4\frac{1}{2}-digit (e.g., 19.999) displays. The 4\frac{1}{2}-digit model costs substantially more. Both units are otherwise identical. The specifications list the uncertainties for three elemental errors:

<table>
<thead>
<tr>
<th>Linearity error</th>
<th>Hysteresis error</th>
<th>Sensitivity error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15% FS0</td>
<td>0.20% FS0</td>
<td>0.25% FS0</td>
</tr>
</tbody>
</table>
For a full-scale output (FSO) of 20 kPa, select a readout based on appropriate uncertainty calculations. Explain.

5.11 The shear modulus, $G$, of an alloy can be determined by measuring the angular twist, $\theta$, resulting from a torque applied to a cylindrical rod made from the alloy. For a rod of radius $R$ and a torque applied at a length $L$ from a fixed end, the modulus is found by $G = 2LT / \pi R^4 \theta$. Examine the effect of the relative uncertainty of each measured variable on the shear modulus. If during test planning all of the uncertainties are set at 1%, what is the uncertainty in $G$?

5.12 An ideal heat engine operates in a cycle and produces work as a result of heat transfer from a thermal reservoir at an elevated temperature $T_h$ and by rejecting energy to a thermal sink at $T_c$. The efficiency for such an ideal cycle, termed a "Carnot cycle," is

$$\eta = 1 - \frac{T_c}{T_h}.$$ 

Determine the required uncertainty in the measurement of temperature to yield an uncertainty in efficiency of 1%. Assume errors are uncorrelated. Use $T_h = 1000$ K and $T_c = 300$ K.

5.13 Heat transfer from a rod of diameter $D$ immersed in a fluid can be described by the Nusselt number, $Nu = hD/k$, where $h$ is the heat-transfer coefficient and $k$ is the thermal conductivity of the fluid. If $h$ can be measured to within $\pm 7\%$ (95%), estimate the uncertainty in $Nu$ for the nominal value of $h = 150$ W/m$^2$·K. Let $D = 20 \pm 0.5$ mm and $k = 0.6 \pm 2\%$ W/m·K.

5.14 Estimate the design-stage uncertainty in determining the voltage drop across an electric heating element. The device has a nominal resistance of $30 \Omega$ and power rating of 500 W. Available is an ohmmeter (accuracy: within 0.5%; resolution: 1 Ω) and ammeter (accuracy: within 0.1%; resolution: 100 mA). Recall $E = IR$.

5.15 Explain the critical difference(s) between a design-stage uncertainty analysis and an advanced-stage uncertainty analysis.

5.16 From an uncertainty analysis perspective, what important information does replication provide that is not found by repetition alone? How is this information included in an uncertainty analysis?

5.17 A displacement transducer has the following specifications:

- **Linearity error:** $\pm 0.25\%$ reading
- **Drift:** $\pm 0.05\%$/$^\circ$C reading
- **Sensitivity error:** $\pm 0.25\%$ reading
- **Excitation:** 10-25 V dc
- **Output:** 0-5 V dc
- **Range:** 0-5 cm

The transducer output is to be indicated on a voltmeter having a stated accuracy of $\pm 0.1\%$ reading with a resolution of 10 μV. The system is to be used at room temperature, which can vary by $\pm 10^\circ$C. Estimate an uncertainty in a nominal displacement of 2 cm at the design stage. Assume 95% confidence.

5.18 The displacement transducer of Problem 5.17 is used in measuring the displacement of a body impacted by a mass. Twenty measurements are made, which yield

$$\bar{x} = 17.20 \text{ mm} \quad s_x = 1.70 \text{ mm}$$

Determine a best estimate for the mass displacement at 95% probability based on all available information.