Defect Reliability Statistics with Redundancy

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Introduction

This paper derives analytical expressions for Sort yield and use-condition reliability fallout as a function of test chip baseline reliability characteristics, yield defect density, die area, and number and area of redundant elements. The expressions take into account the fact that a redundant element used at Sort is not available for redundancy repair in "use". Poisson statistics are assumed for the yield functions, but much of the analysis does not depend on this.

Model of Redundant Chip

We describe a redundant chip by non-repairable and repairable elements. Non-repairable elements will fail if one or more defects fall on them. The model we will consider has a variable number of repairable elements, *n*, each of which can survive 0 or 1 defects.

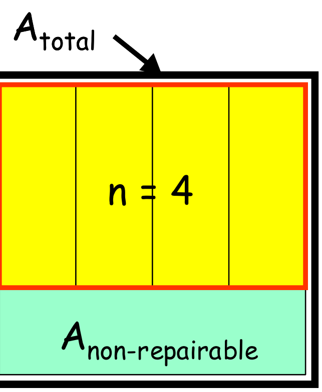

Fig. 1 Conceptual model of redundant chip. Each of the 4 repairable elements can survive one defect.

We could have considered models in which the number of defects which a repairable element can survive is more than 1, and which more precisely reflect circuit architectures. However, the simple description we have chosen uses a minimum number of parameters, and yet describes the main effects of defect reliability when circuit redundancy is available. The variable "*n*" may be regarded as an indicator of the quality of circuit redundancy because, to first order (in the limit of small defect densities), the effect of *n* repairable elements each of which can survive up to one defect is the same as one repairable element which can survive up to *n* defects.

Probability Models

Poisson Models

Using expressions from Poisson statistics derived in the Appendix, we can write down expressions for probabilities defined in Table I.

Table I Definitions of symbols used in analysis.

Symbol	Definition	Model
D	Defect density at sort (cm^{-2})	N/A
D_{bi}	Density of latent reliability defects made visible to test by burn in.	N/A
D_{use}	Density of latent reliability defects made visible to test by burn in and “use”.	N/A
K	Ratio of “reliability” defect density to sort defect density.	Typical value ~ 1%
A_{total}	Total die area.	N/A
r	Fraction of die area which is repairable.	N/A
$A_r = rA_{total}$	Area (cm^2) of repairable part of die. Has *n* redundant elements.	N/A
$A_{nr} = (1-r)A_{total}$	Area (cm^2) of non-repairable part of die.	N/A
n	Number of repairable subelements of area A_r.	N/A
Y_{nr}^0	Probability of 0 defects in non-repairable part of die at sort.	$Y_{nr}^0 = \exp(-A_{nr}D)$
Y_r^0	Probability of 0 defects in a repairable element at sort.	$Y_r^0 = \exp(-A_r D)$
Y_r^1	Probability of exactly 1 defect in a repairable element at sort.	$Y_r^1 = A_r D \exp(-A_r D)$
W_{nr}^0	Prob. of 0 defects in non-repairable part of die activated by burn-in.	$W_{nr}^0 = \exp(-A_{nr} D_{bi})$
W_r^0	Probability of 0 defects in a repairable element activated by burn in.	$W_r^0 = \exp(-A_r D_{bi})$
W_r^1	Prob. of exactly 1 defect in repairable element activated by burn in.	$W_r^1 = A_r D_{bi} \exp(-A_r D_{bi})$
U_{nr}^0	Probability of 0 defects in non-repairable part of die activated by burn-in and “use”.	$U_{nr}^0 = \exp(-A_{nr} D_{use})$
U_r^0	Prob. of 0 defects in repairable element activated by burn in & “use”.	$U_r^0 = \exp(-A_r D_{use})$
U_r^1	Probability of exactly 1 defect in a repairable element activated by burn in and “use”.	$U_r^1 = A_r D_{use} \exp(-A_r D_{use})$

Defect Densities

A technology can be characterized by a defect reliability model. Cumulative fraction failing of a chip without redundancy repair under stress conditions can be modeled as a lognormal distribution:

$$F(t) = \Phi\left[\frac{\ln(t) - \mu}{\sigma}\right] \tag{1}$$

Assuming Poisson statistics of latent reliability defects, the density of latent reliability defects activated by burn in and made detectable by test after t_{bi} hours of burn in is

$$D_{bi} = K \times D \times \frac{\ln\left[1 - \Phi\left(\frac{\ln(t_{bi}) - \mu}{\sigma}\right)\right]}{\ln\left[1 - \Phi\left(\frac{\ln(t = t_{bi\_K}) - \mu}{\sigma}\right)\right]} \tag{2}$$

while after t_{bi} hours of use and t_{use} hours of use the density of activated latent reliability defects is:

$$D_{use} = K \times D \times \frac{\ln\left[1 - \Phi\left(\frac{\ln(t_{bi} + t_{use} / AF) - \mu}{\sigma}\right)\right]}{\ln\left[1 - \Phi\left(\frac{\ln(t = t_{bi\_K}) - \mu}{\sigma}\right)\right]} \tag{3}$$

where AF is the acceleration between burn in and use, where K is the ratio of burn in defect density determined at t_{bi_K} to yield defect density D. (Notice that when $t_{bi} = t_{bi\_K}$, $D_{bi} = K\,D$.) The parameters in Table II would be determined from an experiment using non-redundant SRAMs.

Table II. Example values for latent reliability defect baseline.

Parameter	**Example Values**	**Units**
AF	100 – 300	None
σ	25	ln hours
μ	71	ln hours

Yield Analysis

Sort Yield

The redundant yield formula is obtained by recognizing that the probability of a good die is given by

Probability of a good die =

(Prob. of 0-defect redundant sub-element

or a 1-defect sub-element)$^{\text{Number of repairable sub-elements}}$

and Probability of 0 defects in the non-repairable portion of the die.

That is,

$$Y = [Y_r^0 + Y_r^1]^n Y_{nr} \tag{4}$$

If the Poisson expressions for the probabilities are substituted into this equation, we obtain the special case of Poisson statistics:

$$\begin{aligned} Y &= [\exp(-A_r D) + A_r D \exp(-A_r D)]^n \exp(-A_{nr} D) \\ &= (1 + A_r D)^n \exp[-(A_{nr} + nA_r)D)] \\ &= (1 + A_r D)^n \exp(-A_{total} D) \end{aligned} \tag{5}$$

Burn-In Yield

Study the case of a die surviving sort with repair, and burn-in with repair. Other cases (eg. no repair at sort followed by repair at burn in) can be derived from this. We recognize that

Probability of a good die after burn in =

(Prob. of 0 defects in repairable element after sort

and 0 defects in repairable element after burn-in

or Prob. of 0 defects in repairable element after sort

and 1 defect in repairable element after burn-in

or Prob. of 1 defect in repairable element after sort

and 0 defects in repairable element after burn-in)$^{\text{Number of repairable elements}}$

and Prob. of 0 defects in non-repairable portion of the die at sort

and Prob. of 0 defects in non-repairable portion of the die after burn in.

Or, symbolically

$$W = [Y_r^0 W_r^0 + Y_r^0 W_r^1 + Y_r^1 W_r^0]^n Y_{nr} W_{nr} \tag{6}$$

Eq. (6) is the burn-in yield referred to the pre-sort population. To obtain the yield due to the burn in step by itself, we normalize (6) by (4). That is we calculate the conditional probability (indicated by the prime) of surviving burn in on the condition that the unit has survived sort:

$$W' = W / Y = \left[\frac{Y_r^0 W_r^0 + Y_r^0 W_r^1 + Y_r^1 W_r^0}{Y_r^0 + Y_r^1} \right]^n W_{nr} \tag{7}$$

Substitution into Eq. (7) of the Poisson probability expressions in the above table gives, after some rearrangement:

$$W' = \left(1 + \frac{A_r D_{bi}}{1 + A_r D} \right)^n \exp(-A_{total} D_{bi}) \quad \text{(Repair at sort.)} \tag{8}$$

Eq. (8) is the expression for burn-in yield of a die given that repair has ocurred at sort. The formula for burn-in yield assuming no repair at sort is obtained by taking the limit $KD \to$ Finite, $D \to 0$

$$W' = \left(1 + KA_r D \right)^n \exp(-KA_{total} D) \quad \text{(No repair at sort.)} \tag{9}$$

DPM In Use

By exactly the same arguments, the probability of surviving Sort with repair, and burn in plus "use" with repair is:

$$U = [Y_r^0 U_r^0 + Y_r^0 U_r^1 + Y_r^1 U_r^0]^n Y_{nr} U_{nr} \tag{10}$$

The probability that the die survives burn in plus "use", given that it has survived burn in is

$$U' = U / W = \left(\frac{Y_r^0 U_r^0 + Y_r^0 U_r^1 + Y_r^1 U_r^0}{Y_r^0 W_r^0 + Y_r^0 W_r^1 + Y_r^1 W_r^0} \right)^n \times \frac{U_{nr}}{W_{nr}} \tag{11}$$

The DPM in "use" is therefore

$$F_{use} = 1 - U' \tag{12}$$

When Poisson expressions for survival probabilities are substituted into (11) we find

$$U' = \left[\frac{1 + \dfrac{rA_{total}(D + D_{use})}{n}}{1 + \dfrac{rA_{total}(D + D_{bi})}{n}} \right]^n \times \exp\left[-A_{total} \times (D_{use} - D_{bi}) \right] \tag{13}$$

where D_{use} and D_{bi} are given by Eqs. (2) and (3). When n = 0 (no redundancy) Eq. (13) becomes

$$U' = \exp\left[-A_{total} \times (D_{use} - D_{bi})\right] \tag{14}$$

which corresponds to the survival probability in "use" when the entire area of the chip is a source of latent reliability defects.

If we take the $n \to \infty$ limit of Eq. (13), which corresponds to perfect redundancy, and use the result

$$\left(1 + \frac{x}{n}\right)^n \xrightarrow[n\to\infty]{} \exp(x) \tag{15}$$

we find

$$\begin{aligned} U' &\xrightarrow[n\to\infty]{} \frac{\exp[r \times A_{total}(D + D_{use})]}{\exp[r \times A_{total}(D + D_{bi})]} \times \exp\left[-A_{total} \times (D_{use} - D_{bi})\right] \\ &= \exp[-(1-r) \times A_{total} \times (D_{use} - D_{bi})] = \exp[-A_{nr} \times (D_{use} - D_{bi})] \end{aligned} \tag{16}$$

which corresponds to the survival probability in "use" when only the non-repairable part of the die is a source of latent reliability defects.

Appendix: Poisson Statistics

Failure rate is given by

$$\lambda = AD \tag{A1}$$

Probability of n defects on a die of area A and defect density D is

$$P_n = \frac{\exp(-\lambda)\lambda^n}{n!} \tag{A2}$$

The non redundant failure probability is the probability that 1 or more defects occur on the die:

$$\begin{aligned} F_{\text{non-redundant}} &= \sum_{1}^{\infty} \frac{\exp(-\lambda)\lambda^n}{n!} = \exp(-\lambda)\left(\sum_{0}^{\infty} \frac{\lambda^n}{n!} - 1\right) \\ &= \exp(-\lambda)[\exp(\lambda) - 1] = 1 - \exp(-\lambda) \end{aligned} \tag{A3}$$

So the non-redundant survival probability (the non-redundant yield) is

$$Y_{\text{non-redundant}} = 1 - F_{\text{non-redundant}} = \exp(-\lambda) = \exp(-AD)\,. \tag{A4}$$

For the case of a die of area A that can be repaired if there is 1 defect, the redundant failure probability is the probability that 2 or more defects occur on the die:

$$\begin{aligned} F_{\text{redundant}} &= \sum_{2}^{\infty} \frac{\exp(-\lambda)\lambda^n}{n!} = \exp(-\lambda)\left(\sum_{0}^{\infty} \frac{\lambda^n}{n!} - 1 - \lambda\right) \\ &= \exp(-\lambda)[\exp(\lambda) - 1 - \lambda] = 1 - (1 + \lambda)\exp(-\lambda) \end{aligned} \tag{A5}$$

So the redundant survival probability (the redundant yield) is

$$Y_{\text{redundant}} = 1 - F_{\text{redundant}} = (1+\lambda)\times\exp(-\lambda) = (1+AD)\times\exp(-AD) \qquad \text{(A6)}$$

The redundant yield is the probability that a die has 0 or 1 defects on it. The probability that the die has 0 defects is

$$Y^0 = \exp(-\lambda) = \exp(-AD) \qquad \text{(A7)}$$

And the probability that the die has *exactly* one defect is

$$Y^1 = \lambda\exp(-\lambda) = AD\exp(-AD)\,. \qquad \text{(A8)}$$

The expressions in Eqs. (A7) and (A8) will be used in the body of the document.

In general, the probability that a die has exactly n defects is

$$Y^n = \frac{\lambda^n}{n!}\exp(-\lambda) = \frac{(AD)^n}{n!}\times\exp(-\lambda) \qquad \text{(A9)}$$