RTN Time-in-State Model[1]

Alternative 1. Continuous Test

Consider a variable retention time bit which has an RTN waveform alternating between a high (Hi, +) and a low (Lo, -) state. In the case of a continuous test of duration Δt, if a bit is in the high state at the beginning of the test, then the test can detect any transition to the low state that occurs within the duration of the test. The probability that the test will detect only the high state of the bit is

$$\begin{aligned} &P\left(\text{Continuous test with duration } \Delta t \text{ starting at } t_0 \text{ detects only the Hi state}\right) \\ &\qquad = P\left(\text{Hi during interval } \Delta t \text{ starting at } t_0 \mid \text{Hi at } t_0\right) P\left(\text{Hi at } t_0\right) \end{aligned} \tag{1}$$

where the probability of finding a bit in a Hi state is

$$P\left(\text{Hi at } t_0\right) = \frac{\tau_+}{\tau_+ + \tau_-} \equiv s \quad . \tag{2}$$

Since this is a Poisson process the distribution of times until the next transition to low is

$$P\left(\text{Hi during interval } \Delta t \text{ starting at } t_0 \mid \text{Hi at } t_0\right) = \exp\left(-\frac{\Delta t}{\tau_+}\right) \tag{3}$$

which is simply the distribution of durations in the high (+) state. Therefore

$$\begin{aligned} &P\left(\text{Measuring only Hi state for test with duration } \Delta t \text{ starting at } t_0.\right) \\ &\qquad = s \exp\left(-\frac{\Delta t}{\tau_+}\right) \end{aligned} \tag{4}$$

Alternative 2. Two Tests, Separated by *Δt.*

This alternative may be modeled using the autocorrelation function for RTN. The autocorrelation function for RTN is $P(+,t|+)$, the probability that the bit is Hi at time t, given that the bit is Hi at time 0. That is $P(+,t|+)$ is the probability that there are an even number of transitions between these times (including 0 transitions). Also, let $P(-,t|+)$, be the probability that the bit is Lo at time t, given that the bit is Hi at time 0. That is, $P(-,t|+)$ is the probability that there are an odd number of transitions between these times. For RTN

$$P_{11}\left(+,t \mid +\right) + P\left(-,t \mid +\right) = 1 \tag{5}$$

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[1] Glenn Shirley, December 2012

and

$$P(+,t+dt\,|\,+)=P(-,t\,|\,+)\frac{dt}{\tau_-}+P(+,t\,|\,+)\left(1-\frac{dt}{\tau_+}\right). \tag{6}$$

Eq. (6) expresses the fact that the change of the $P(+|+)$ probability in the interval t to $t + dt$ is the sum of transitions into the + state from the – state at a constant rate of $1/\tau_-$, and out of the + state at a constant rate of $1/\tau_+$. The following differential equation follows from Eqs. (5) and (6)

$$\frac{dP\left(+,t\,|\,+\right)}{dt}+P\left(+,t\,|\,+\right)\left(\frac{1}{\tau_+}+\frac{1}{\tau_-}\right)=\frac{1}{\tau_-} \tag{7}$$

which may be solved to give

$$P\left(+,t\,|\,+\right)=s+(1-s)\exp\left(-t/\hat{\tau}\right) \tag{8}$$

where

$$s=\frac{\tau_+}{\tau_++\tau_-}\qquad \frac{1}{\hat{\tau}}=\frac{1}{\tau_+}+\frac{1}{\tau_-} \tag{9}$$

In the DRAM test application the retention time waveform is tested instantaneously only at t_0 and at $t_0 + \Delta t$, admitting the possibility that the waveform will transition to the low state any number of times between the measured high states. Using Eq. (8), the probability of finding the bit in the high state at $t_0 + \Delta t$ given that it is in the high state at t_0 is:

$$\begin{aligned}&P\left(\text{Measuring Hi state instantaneously at } t_0+\Delta t\,|\,\text{Hi at } t_0\right)\\&\qquad=s+(1-s)\exp\left(-\Delta t/\hat{\tau}\right).\end{aligned} \tag{10}$$

So

$$\begin{aligned}&P\left(\text{Two instantaneous tests at } t_0 \text{ and } t_0+\Delta t \text{ detects only the Hi state}\right)\\&\quad=P\left(\text{Measuring Hi state instantaneously at } t_0+\Delta t\,|\,\text{Hi at } t_0\right)P\left(\text{Hi at } t_0\right)\\&\quad=s^2+s(1-s)\exp\left(-\Delta t/\hat{\tau}\right)\end{aligned} \tag{11}$$

Alternative 3. Multiple Tests, Separated by Δt_1, Δt_2, ..

In this case the retention time waveform is tested instantaneously at t_0, $t_1 = t_0 + \Delta t_1$, $t_2 = t_0 + \Delta t_1 + \Delta t_2$,.. This admits the possibility that the waveform will transition to the low state any number of times in between the measured high states. The probability that all measurements find the bit in a high state, given that it is in a high state at t_0 is

$$\begin{aligned}
&P\left(\text{Measuring Hi state instantaneously at all of } \{t_i\}_{i=1,n} \mid \text{Hi at } t_0\right)\\
&\qquad =\prod_{i=1}^{n} P\left(\text{Measuring Hi state instantaneously at } t_i \mid \text{Hi at } t_{i-1}\right)\\
&\qquad =\prod_{i=1}^{n}\left[s+(1-s)\exp\left(-\Delta t_i/\hat{\tau}\right)\right]
\end{aligned}\tag{12}$$

So the probability that instantaneous tests at all of $\{t_i\}_{i\,=\,0,n}$ (at $n + 1$ instants) will find the bit only in the high state is

$$\begin{aligned}
&P\left(\text{Measuring Hi state instantaneously at all of } \{t_i\}_{i=0,n}\right)\\
&\qquad = P\left(\text{Measuring Hi state instantaneously at all of } \{t_i\}_{i=1,n} \mid \text{Hi at } t_0\right)P\left(\text{Hi at } t_0\right)\\
&\qquad = s\prod_{i=1}^{n}\left[s+(1-s)\exp\left(-\Delta t_i/\hat{\tau}\right)\right]
\end{aligned}\tag{13}$$

Example 1.

Suppose that $N + 1$ tests are evenly spaced and that the separation between tests goes to zero so that it becomes a continuous test, with a total duration Δt. That is the separation of tests is $\Delta t/N$ as $N \to \infty$. In this case Eq. (13) becomes

$$\begin{aligned}
&P\left(\text{Measuring Hi state instantaneously at all of } \{t_i\}_{i=1,\infty}\right)\\
&\qquad = \lim_{N\to\infty} s\left\{s+(1-s)\exp\left[-\frac{1}{\hat{\tau}}\frac{\Delta t}{N}\right]\right\}^N \simeq \lim_{N\to\infty} s\left\{s+(1-s)\left(1-\frac{1}{\hat{\tau}}\frac{\Delta t}{N}\right)\right\}^N\\
&\qquad \simeq \lim_{N\to\infty} s\left(1-\frac{1-s}{\hat{\tau}}\frac{\Delta t}{N}\right)^N = \lim_{N\to\infty} s\left(1-\frac{\tau_+ + \tau_-}{\tau_+\tau_-}\frac{\tau_-}{\tau_+ + \tau_-}\frac{\Delta t}{N}\right)^N\\
&\qquad \simeq \lim_{N\to\infty} s\left\{1-\frac{\Delta t/\tau_+}{N}\right\}^N = s\exp\left(-\frac{\Delta t}{\tau_+}\right)
\end{aligned}\tag{14}$$

So the limiting case of Eq. (14) recovers Eq. (4) as it must.

Example 2.

Suppose there are two instantaneous tests separated by Δt. From Eq. (11) the probability that both tests find the bit in the high state is

$$P\left(\text{Both bits high.}\right) = s^2 + s(1-s)\exp\left(-\Delta t/\hat{\tau}\right) \tag{15}$$

and by symmetry the probability that both tests find the bits in the low state is

$$P\left(\text{Both bits low.}\right) = (1-s)^2 + s(1-s)\exp\left(-\Delta t/\hat{\tau}\right) . \tag{16}$$

Since

$$P\left(\text{Both bits high.}\right) + P\left(\text{Both bits low.}\right) + P\left(\text{One bit high and the other low.}\right) = 1 \tag{17}$$

then

$$P\left(\text{One bit high and the other low.}\right) = 2s(1-s)\left[1-\exp\left(-\Delta t/\hat{\tau}\right)\right] \tag{18}$$

The high/low probability is irrespective of whether the first test or the second test finds the bit high.